Commun. Comput. Phys.
doi: 10.4208/cicp.201109.261110s

Vol. **11**, No. 2, pp. 285-302
February 2012

Perfectly Matched Layer with Mixed Spectral Elements for the Propagation of Linearized Water Waves

Gary Cohen and Sébastien Imperiale*

INRIA, Domaine de Voluceau, Rocquencourt-BP 105, 78153 Le Chesnay Cedex, France.

Received 20 November 2009; Accepted (in revised version) 26 November 2010

Available online 24 October 2011

Abstract. After setting a mixed formulation for the propagation of linearized water waves problem, we define its spectral element approximation. Then, in order to take into account unbounded domains, we construct absorbing perfectly matched layer for the problem. We approximate these perfectly matched layer by mixed spectral elements and show their stability using the "frozen coefficient" technique. Finally, numerical results will prove the efficiency of the perfectly matched layer compared to classical absorbing boundary conditions.

AMS subject classifications: 35J05

PACS: 41.20.cv, 43.20.Hq

Key words: Linearized water waves, perfectly matched layer, mixed finite elements.

1 Introduction

In this article we focus on the water wave equations presented in [1] which model gravity wave generation and propagation in water, in its complete form, it read as solving a homogeneous Laplacian problem in water coupled with a non linear boundary condition on the surface, depending on time. A simpler model is obtained by linearizing the surface condition in order to only describe the propagation of the gravity wave, which is sufficient for waves of small amplitudes compared to the deepness of the bottom and the wavelength. The purpose of this paper is to develop original absorbing perfectly matched layers (PML) to take into account the propagation of linearized water waves (LWW) problem in unbounded domains and to present their finite element discretization. PML was introduced by Bérenger [2] for hyperbolic problems. In this paper we extend these ideas to a strongly elliptic problem. We will present the approximation of

*Corresponding author. *Email addresses:* `gary.cohen@inria.fr` (G. Cohen), `sebastien.imperiale@inria.fr` (S. Imperiale)

the PML for the LWW equations by high order mixed spectral elements with Legendre-Gauss-Lobatto points. This approach was successfully applied to the acoustics and linear elastodynamic equations [3, 4]. For classical transient equations, this approximation is fundamental, as it provides mass-lumping which substantially reduces the cost of the method. In our case, the mixed form of the spectral element method enables a simple writing of the PML system, without adding extra variables (as in [2] and [5]), and leads to a low storage and factorization of the propagation operator (the stiffness matrix). This second property provides an efficient algorithm for wave equations in the frequency domain [6]. This factorization can be even more efficient for a Laplacian problem, which justifies our approximation for the LWW problem. The stability of our PML is also studied. Whereas stability issues are often a difficult question (especially for elastodynamic waves), in this article we manage to prove the stability of the PML at a continuous level. We also present 2D numerical results that show that previously designed high order absorbing condition (see [7]) is long time unstable, whereas the PML are long time stable at a discrete level.

Our paper is divided into four parts: in the second section, we introduce the mixed form of the problem and its variational formulation. In the third section, we construct its approximation by spectral elements whose principle is recalled. Thereafter, we present the discrete formulation by pointing out the sparse and low storage character of the matrices involved. In the fourth section, after discussing the stability of the absorbing boundary conditions (ABC) of first order for taking into account unbounded domains [7], we construct perfectly matched layers using the reformulation introduced by Chew and Weedon [5]. We then show how to apply the mixed spectral element method to these PML. Lastly we show the stability of the continuous PML by using a frozen coefficient technique as in [8]. The fifth section is devoted to numerical experiments which prove the stability and the efficiency in terms of reflections of the PML compared to the first order ABC.

2 The continuous problem

2.1 Classical formulation

Let Ω be an open domain of $\mathbb{R}^d$ $(d=2,3)$ and Γ_S, Γ_B and Γ_h three subsets providing a partition of $\partial\Omega$.