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Numerical Regularized Moment Method for High Mach Number Flow

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Abstract. This paper is a continuation of our earlier work [SIAM J. Sci. Comput., 32(2010), pp. 2875–2907] in which a numerical moment method with arbitrary order of moments was presented. However, the computation may break down during the calculation of the structure of a shock wave with Mach number $M_0 \geqslant 3$. In this paper, we concentrate on the regularization of the moment systems. First, we apply the Maxwell iteration to the infinite moment system and determine the magnitude of each moment with respect to the Knudsen number. After that, we obtain the approximation of high order moments and close the moment systems by dropping some high-order terms. Linearization is then performed to obtain a very simple regularization term, thus it is very convenient for numerical implementation. To validate the new regularization, the shock structures of low order systems are computed with different shock Mach numbers.

AMS subject classifications: 65M08, 65M12

Key words: Boltzmann-BGK equation, Maxwellian iteration, regularized moment equations.

1 Introduction

In the field such as high altitude flight and microscopic flows, gas is considered to be very rarefied and outside the hydrodynamic regime. In this case, usual fluid models such as Euler equations and Navier-Stokes-Fourier system will fail when the rarefied effect is significant. The moment method, which was first proposed by Grad [6], is focused on the description of the rarefied gases using a small number of variables. Almost all moment methods are derived from the Boltzmann equation which is regarded to be able to capture the rarefied effects accurately. In [4], a special expansion of the distribution functions

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is adopted to make it possible to solve the associated Grad-type moment equations numerically without the explicit expressions of the system, and then the numerical scheme was regularized using the technique of a modified Chapman-Enskog expansion following [16]. In [4], it has been verified numerically that a smooth shock structure with Mach number $M_0 = 2$ can be obtained by solving the R20 equations with a Riemann problem until the steady state. However, it was found in our numerical experiments that if we set the shock Mach number $M_0 \geqslant 3$, the computation will break down with negative temperature appearing inside the shock wave before a steady structure of the shock wave is formed, which is possibly caused by the non-hyperbolic nature of the moment system.

In this paper, we present a new regularization method which is able to produce smooth profile for large Mach number shock waves and low order moment systems. The idea originates from the order-of-magnitude method [13, 15], where the order of magnitude for each moment with respect to the Knudsen number is investigated in order to obtain a transport system with a specified order of accuracy. Additionally, from the computational perspective, a conservative form of moment equations is preferred, so we put this idea into the framework of [4] and derive a uniform expression of the regularization terms for all moment systems.

As the first step, we derive the analytical form of the moment equations using the same set of moments as in [4]. Once the moment equations are given explicitly, we find that only conservative variables and the moments within five successive orders appear in each equation. With the help of Maxwellian iteration [8], the order of magnitude can be obtained for each moment, and this skill has been used in [10]. The closure of the moment system is achieved using a similar skill as in [10,14]. We approximate the $(M+1)$-st order moments by removing all terms with higher orders of magnitude than the leading order in the corresponding equation to get a closed system of all moments with orders lower than M. Eventually, a parabolic system is explicitly obtained.

The resulting regularization term is somewhat complicated and is simplified using the technique of linearization for the sake of convenient numerical implementation. As in [16], the fluid is considered to be in the vicinity of velocity-free equilibrium states, thus the derivatives are small. Dropping the terms which are nonlinear in small values, the remaining linear part turns out to be very compact. In the 1D case, it is obvious that the regularization introduces additional diffusion to the M-th order moments. With the simplified regularization term, the numerical investigation of the shock tube problem shows the convergence to the Boltzmann-BGK equation in moments. And it is numerically demonstrated that smooth shock profiles can be obtained for large Mach numbers.

The layout of this paper is as follows: in Section 2, an overview of Boltzmann-BGK model and our discretization of the distribution function is introduced as some preliminaries. In Section 3, the details of the new regularization method are presented. In Section 4, we present two numerical examples to make comparisons between results for different moment equations, different Knudsen numbers and different Mach numbers. At last, some concluding remarks will be given in Section 5.