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Micro-Differential Boundary Conditions Modelling the Absorption of Acoustic Waves by 2D Arbitrarily-Shaped Convex Surfaces

Hélène Barucq[1,2], Julien Diaz[1,2] and Véronique Duprat[2,1,*]

[1] *INRIA Bordeaux-Sud Ouest, Team-project MAGIQUE-3D.*
[2] *LMA, CNRS UMR 5142, University of Pau, France.*

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Abstract. We propose a new Absorbing Boundary Condition (ABC) for the acoustic wave equation which is derived from a micro-local diagonalization process formerly defined by M.E. Taylor and which does not depend on the geometry of the surface bearing the ABC. By considering the principal symbol of the wave equation both in the hyperbolic and the elliptic regions, we show that a second-order ABC can be constructed as the combination of an existing first-order ABC and a Fourier-Robin condition. We compare the new ABC with other ABCs and we show that it performs well in simple configurations and that it improves the accuracy of the numerical solution without increasing the computational burden.

AMS subject classifications: 35L05

Key words: Wave equation, micro-local diagonalization, absorbing boundary condition, finite element formulation.

1 Introduction

The numerical simulation of waves propagation generally involves boundary conditions which both represent the behavior of waves at infinity and provide a mathematical tool to define a bounded computational domain in which a finite element method can be applied. Most of these conditions can be justified as an approximation of the Dirichlet-to-Neumann operator and when they both preserve the sparsity of the finite element matrix and enforce dissipation into the system, they are called Absorbing Boundary Conditions (ABC). Obviously an ABC impacts the accuracy of the numerical solution which can be improved by using high-order conditions. Nevertheless a high-order condition

*Corresponding author. *Email addresses:* `helene.barucq@inria.fr` (H. Barucq), `julien.diaz@inria.fr` (J. Diaz), `veronique.duprat@univ-pau.fr` (V. Duprat)

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requires to introduce auxiliary unknowns to be easily employed in a finite element formulation [12–14] which increases the computational cost significantly. Moreover, increasing the order of the condition can destroy the long-time stability of the wave equation. Another limitation of existing ABCs is the type of surface bearing the ABC. Most of the high-order ABCs have been derived for flat surfaces. They can be used for more general boundaries which can be described by a collection of segments. Nevertheless, it is necessary to introduce matching conditions between each segments which can be cumbersome to implement.

In this paper, we show how to construct an ABC that fits the following criteria:

1. the ABC can be apply to arbitrarily-shaped boundaries;
2. the ABC does not require significant additional computations to be handled in a finite element formulation;
3. the ABC preserves the long-time stability of the wave equation.

For that purpose, we investigate the possibility of improving an existing first-order ABC which can be applied on arbitrarily-shaped boundaries. To minimize the computational cost, we limit our study to the construction of second-order differential conditions. Regardless of the implementation aspect, we can also note that Engquist and Majda showed that conditions of order greater than two could lead to an ill-posed problem. The existing condition that we use is the first-order condition involving the curvature of the absorbing boundary [10]. This condition is very easy to include in a variational formulation. To justify our choice, we begin with comparing the curvature condition with the BGT2 condition, introduced by Bayliss, Gunzburger and Turkel in [5], extended to the time domain, knowing that the BGT2 condition is widely used by engineers. We show that the extended BGT2 condition requires to introduce an auxiliary unknown and that it performs as well as the curvature condition. We then investigate how to improve the performances of the curvature condition. For that purpose, we use a generalization of the Taylor diagonalization process by considering the principal symbol of the wave equation both in the hyperbolic and the elliptic regions. Our approach meets an idea that was formerly investigated in [12–14] but for flat surfaces extending conditions formerly proposed by Higdon [15]. By using a classical finite element scheme, Hagstrom *et al.* [12–14] have shown the improvements induced by the new condition. Nevertheless, we have observed that the Hagstrom *et al.* condition is unstable when employed in an Interior Penalty Discontinuous Galerkin (IPDG) formulation while our new condition seems to be stable. Moreover, the Hagstrom *et al.* condition seems to be difficult to use on curved surfaces while our condition can be applied straightforwardly, without any corner condition. The new condition that we construct is the combination of two boundary conditions which each leads to a long-time stable system. Hence the new ABC should outperform the Hagstrom *et al.* condition.

In this paper, we consider a model problem for the time-dependent wave equation in a two-dimensional domain Ω with an obstacle inside and an ABC on its external bound-