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A Low Frequency Model for Acoustic Propagation in a 2D Flow Duct: Numerical Computation

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Abstract. In this paper we study a low frequency model for acoustic propagation in a 2D flow duct. For some Mach profile flow, we are able to give a well-posedness theorem. Its proof relies on a quasi-explicit expression of the solution which provides us an efficient numerical method. We give and comment numerical results for particular linear, tangent and quadratic profiles. Finally, we give a numerical validation of our asymptotic model.

AMS subject classifications: 35B35, 35Q35, 35C15

Key words: Acoustics in flow, Galbrun's equations, asymptotic analysis, Laplace transform.

1 Introduction

The present work has been motivated by applications to computational aeroacoustics, namely the numerical modelling of the propagation of sound in a moving fluid. In this perspective, we have chosen to reconsider the problem from a fundamental point of view and to begin with the propagation of sound in a duct. We consider a strongly varying parallel flow which can be simply described with the help of a scalar function $M(y)$ which represents, after appropriate normalisation (it is the Mach number), the lateral variations of the velocity of the reference flow. We shall call this function the Mach profile. For this model problem, a quasi-1D mathematical problem has been obtained in [1] from Galbrun's equations [5] (which are equivalent to the well-known linearized Euler equations) by a formal asymptotic expansion with respect to the width of the tube (this can also be

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seen as a low frequency analysis). This model leads to the following initial value problem, where the unknown u is the limit (as the width tends to 0) of the x component of the Lagrange displacement:

$$\begin{cases} \text{Find } u(x,y,t): \mathbb{R}\times[-1,1]\times\mathbb{R}^+ \to \mathbb{R}, \\ \left(\dfrac{\partial}{\partial t}+M(y)\dfrac{\partial}{\partial x}\right)^2 u-\dfrac{\partial^2}{\partial x^2}Eu=0, \quad (x,y)\in\mathbb{R}\times[-1,1], \quad t>0, \\ u(x,y,0)=u_0(x,y), \quad (x,y)\in\mathbb{R}\times[-1,1], \\ \dfrac{\partial u}{\partial t}(x,y,0)=u_1(x,y), \quad (x,y)\in\mathbb{R}\times[-1,1], \end{cases} \tag{1.1}$$

where $M\in L^\infty(]-1,1[)$ is the Mach profile and E is the following averaging operator on $L^2(]-1,1[)$:

$$E: u \to Eu=\frac{1}{2}\int_{-1}^{1} u(y)dy. \tag{1.2}$$

This model is local (differential) in x, the coordinate along the axis of the tube, but non local in y, the transversal coordinate. The paper [2] is devoted to the mathematical analysis of this problem, using the Fourier transform and spectral theory. Despite its apparent simplicity this problem has rather surprising properties. In particular, we exhibit in [2] a necessary condition for the problem to be well-posed and it was conjectured that it is a sufficient condition. The authors of [6] are able to show this conjecture for a class of smooth monotonous and convex (or concave) profiles. This was done by calculating a quasi-explicit expression of the solution, using the Fourier-Laplace transform. This present paper is devoted to the numerical computation of the solution of (1.1), hence it is based on the two articles [2] and [6]. In Section 2, after reminding some properties of (1.1), we give a new well-posedness theorem (2.1) based on the method used in [6]. Then we apply this theorem to the case of a class of smooth monotonous profiles for which more calculations can be done, to get Theorem 2.2. After that, the case of the quadratic profile $M(y)=1-y^2$ is explored, it constitutes an example of extension to non monotonous profiles. The proof of Theorem 2.1 relies on a quasi-explicit representation of the solution which is given; this representation of the solution will provide an efficient way to compute the solution. The numerical method is presented in Section 3 and is then illustrated, for smooth monotonous profiles, by the linear and tangent profiles, and for a non monotonous profile, by the quadratic profile $M(y)=1-y^2$. In the last section, we give a numerical validation of our asymptotic model by comparison to a 2D Eulerian code.

At this stage of the work, our contribution remains quite academic and several steps need to be done before asserting that this kind of approximate model is useful for the applications.

- Because of its non-local nature, our model is not easily tractable from the numerical point of view. However approximating the reference profile $M(y)$ by a piecewise