

Complete Radiation Boundary Conditions for Convective Waves

Thomas Hagstrom^{1,*}, Eliane Bécache², Dan Givoli³ and Kurt Stein¹

¹ Dept. of Mathematics, Southern Methodist University, Dallas, TX 75275, USA.

² Propagation des Ondes, Etude Mathématique et Simulation (POEMS), INRIA, Domaine de Voluceau-Rocquencourt, BP 105, 78153 Le Chesnay Cedex, France.

³ Dept. of Aerospace Engineering, Technion, Haifa 32000, Israel.

Received 23 December 2009; Accepted (in revised version) 6 January 2011

Available online 24 October 2011

Abstract. Local approximate radiation boundary conditions of optimal efficiency for the convective wave equation and the linearized Euler equations in waveguide geometry are formulated, analyzed, and tested. The results extend and improve for the convective case the general formulation of high-order local radiation boundary condition sequences for anisotropic scalar equations developed in [4].

AMS subject classifications: 65M12, 35L50, 35C15

Key words: Radiation boundary conditions, convective waves.

1 Introduction

The problem of imposing accurate and efficient radiation boundary conditions at non-physical boundaries is central in the numerical analysis of wave propagation problems. For isotropic systems, recent results [14] provide local radiation boundary condition sequences which guarantee any desired accuracy using a minimal number of terms; precisely the complexity of the procedure scales as the logarithm of the error tolerance multiplied by the logarithm of the dimensionless parameter cT/δ where T is the simulation time, c is the wavespeed, and δ is the minimal separation between wave sources and the boundary. The goal of this paper is to extend these results to convective waves.

Low order local radiation boundary conditions for convective waves have been used for at least thirty years [5, 16]. The first application of high-order conditions we know of,

*Corresponding author. *Email addresses:* thagstrom@smu.edu (T. Hagstrom), eliane.becache@inria.fr (E. Bécache), givolid@aerodyne.technion.ac.il (D. Givoli), kcstein@smu.edu (K. Stein)

however, came much later [13]. As advocated here, the implementation is based on auxiliary functions. The details of our current approach for general anisotropic scalar wave equations are given in [4]. What is new in this work is the combination of the general formulation in [4] with the optimal complete radiation boundary condition parameters derived in [14], the presentation of numerical experiments for convective problems, and the generalization of the construction to the linearized Euler equations. Here one must deal with the presence of vortical modes, which requires small changes to the boundary condition formulation.

We note that other accurate methods do exist for convective waves. Exact nonlocal conditions have been implemented for the linearized Euler equations in [1, 2, 10]; these are certainly accurate, but more costly and less flexible than the approach suggested here. Another approach is based on the so-called perfectly matched layer (PML). Although original formulations of PML for the linearized Euler equations were unstable [3], it was soon discovered how they could be stabilized [6, 9, 15]. As mentioned in [4], our boundary condition formulation can be interpreted as a nonstandard semidiscretized PML. However, for long time computations it is more efficient than the standard approach, as then the layer thickness must grow like \sqrt{T} . See, for example, the exact error analysis given by Diaz and Joly [7]. In any case, our method has the advantage of providing a parametrization with any prescribed accuracy without the need to tune ad hoc absorption or grid stretching profiles.

2 The convective wave equation

We first consider the equation

$$\left(\frac{\partial}{\partial t} + V_x \frac{\partial}{\partial x}\right)^2 u = c^2 \nabla^2 u + f, \tag{2.1}$$

where, for definiteness, we assume a waveguide geometry

$$(x, y) \in \mathbb{R} \times \Omega, \quad \alpha \frac{\partial u}{\partial n} + \beta u = 0, \quad y \in \partial\Omega, \tag{2.2}$$

a subsonic, rightmoving flow

$$0 < M_x \equiv \frac{V_x}{c} < 1, \tag{2.3}$$

and data, $u(x, y, 0)$, $\frac{\partial u}{\partial t}(x, y, 0)$, $f(x, y, t)$ supported in $(-L, L) \times \Omega$. Here $\Omega \subset \mathbb{R}^d$ in general, though our numerical experiments will be confined to $d = 1$ and $\Omega = (-1, 1)$. Also we assume that c , V_x and, therefore, M_x are constant. Our goal is to construct and test accurate, efficient radiation boundary conditions at $x_R = \pm(L + \delta)$ for δ small. A general theory of high-order radiation conditions for anisotropic and convective wave equations is developed in [4]. Here we combine that theory with the optimal parametrizations of [14], which we call complete radiation boundary conditions (CRBCs).