

Impedance Boundary Condition for Lattice Boltzmann Model

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Abstract. A surface based lattice Boltzmann impedance boundary condition (BC) using Ozyoruk's model [J. Comput. Phys., 146 (1998), pp. 29-57] is proposed and implemented in PowerFLOW. In Ozyoruk's model, pressure fluctuation is directly linked to normal velocity on an impedance surface. In the present study, the relation between pressure and normal velocity is realized precisely by imposing a mass flux on the surface. This impedance BC is generalized and can handle complex geometry. Combined with the turbulence model in the lattice Boltzmann solver PowerFLOW, this BC can be used to model the effect of a liner in presence of a complex 3D turbulent flow. Preliminary simulations of the NASA Langley grazing flow tube and Kundt tube show satisfying agreement with experimental results.

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1 Introduction

Sound absorbing materials are widely used in various industries to reduce noise emission. For example in modern turbofan engines, the inlet wall is treated with acoustic liners. Highway and railway noise barriers often use acoustic treatments for reducing community noise issues. These sound absorbing materials are composed of porous media allowing non-zero normal velocity at the surface. Considering the difficulty to model the flow and the acoustic propagation inside porous media, an acoustic liners are usually handled with macroscopic boundary conditions imposed in the frequency domain, consisting of a quantity called impedance, defined as

$$\hat{p}(\mathbf{x}, \omega) = Z(\omega) \hat{\mathbf{u}}(\mathbf{x}, \omega) \cdot \mathbf{n}(\mathbf{x}), \quad (1.1)$$

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where \hat{p} is the acoustic pressure, $\hat{\mathbf{u}}$ is the acoustic velocity and \mathbf{n} is the mean surface normal. The impedance is a frequency-dependent complex quantity given by

$$Z(\omega) = R(\omega) + iX(\omega), \quad (1.2)$$

where $R(\omega)$ and $X(\omega)$ are the resistance and reactance, respectively, of the liner. These are properties of the material and can be measured experimentally in a Kundt tube. The impedance Z given by Eq. (1.1) provides a relation between acoustic pressure and velocity at the surface in the frequency domain. In order to be used as a boundary condition in the physical domain of a time-explicit computational fluid dynamics (CFD) simulation, this relation has to be transformed into time domain. Mathematically, the time-domain equivalent of the frequency domain impedance condition can be derived by taking the inverse Fourier transform. However, due to the convolution integral, long time history of the acoustic velocity would be required. Ozyoruk et al. [1] proposed an efficient implementation of the above impedance boundary condition in the time domain by using the z -transform. Taking advantage of the time-shifting and convolution properties of the z -transform, the implementation only needs to store values of pressure and velocity at a few previous time steps. The simulations of the NASA Langley grazing flow tube case using this model [1–3] showed good agreement with experimental data. Toutant et al. [4] applied this scheme within a lattice Boltzmann method (LBM) flow solver and also showed similarly good correlations to the experiment. However, this implementation of the impedance boundary condition is limited to a wall boundary perfectly aligned with the cell boundaries of the underlying LBM grid. Hence Toutant's approach is not suitable for treating complex geometries with inclined or curved boundaries.

In the present paper, we extend the previous work to provide a surface based impedance BC for arbitrary wall boundary geometry which incorporates Ozyoruk's model in a generalized LBM flow solver PowerFLOW. The effective LB boundary treatment in PowerFLOW provides the ability to impose desired boundary conditions, for example frictionless BC, on complex geometries [5]. The advanced Very-Large-Eddy-Simulation (VLES) turbulence model applied in PowerFLOW [6,7] further enables accurate simulations of high Reynolds number turbulent flows. Therefore realization of the generalized time-domain impedance BC in PowerFLOW makes possible for the first time quantitative analyzes of more realistic aero-acoustic problems besides acoustic liners represented by Ozyoruk's model.

2 Impedance boundary condition

For the constant depth ceramic tubular liner (CT73) [8] the following impedance function is proposed in [1] by curve fitting the experimental data:

$$\frac{Z(\omega)}{\rho_0 c_0} = r_1 + \frac{r_2 - r_1}{1 + i\omega r_3} + \frac{i\omega r_4}{(1 - \omega^2/r_6^2) + i\omega r_5} + i\omega r_7, \quad (2.1)$$