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Time-Harmonic Acoustic Scattering in a Complex Flow: A Full Coupling Between Acoustics and Hydrodynamics

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Abstract. For the numerical simulation of time harmonic acoustic scattering in a complex geometry, in presence of an arbitrary mean flow, the main difficulty is the coexistence and the coupling of two very different phenomena: acoustic propagation and convection of vortices. We consider a linearized formulation coupling an augmented Galbrun equation (for the perturbation of displacement) with a time harmonic convection equation (for the vortices). We first establish the well-posedness of this time harmonic convection equation in the appropriate mathematical framework. Then the complete problem, with Perfectly Matched Layers at the artificial boundaries, is proved to be coercive + compact, and a hybrid numerical method for the solution is proposed, coupling finite elements for the Galbrun equation and a Discontinuous Galerkin scheme for the convection equation. Finally a 2D numerical result shows the efficiency of the method.

AMS subject classifications: 35Q35, 65N30

Key words: Aeroacoustics, scattering of sound in flows, Galbrun equation, advection equation, finite elements, discontinuous Galerkin method.

1 Introduction

The reduction of noise is becoming today a main objective whose progress is, in particular, related to a better understanding of the complex phenomena occurring when acoustic waves propagate in presence of a mean flow. For instance, the radiation of the sound produced by aircraft engines is strongly influenced by the presence of the flow around the

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airplane. Several methods have been developed to solve the time-domain Linearized Euler Equations, but the treatment of the artificial boundaries still raises open questions. On the other hand, the time-harmonic problem has been considered only in the simplest case of a potential mean flow, apart for some attempts to solve the model of Galbrun in a general flow [9]. Galbrun's system corresponds to a linearized model whose unknown $\mathbf{u}$ is the perturbation of the Lagrangian displacement. It results in second order equations in time and in space, at first sight similar to more classical wave models. Contrary to the Linearized Euler Equations, Galbrun's system does not involve any derivatives of the mean flow quantities.

Our objective is to develop a numerical method to solve time-harmonic Galbrun's system, in a quite general case in the sense that the geometry, and therefore the mean flow, can be complex. As a consequence, discretization methods written on an unstructured mesh will be privileged. It is now well-known that a direct resolution using finite elements combined with Perfectly Matched Layers does not work. Extending an approach originally applied to time-harmonic Maxwell equations, we have shown that the difficulties can be overcome by writing a so-called augmented equation. This augmented equation requires the evaluation of

$$\psi = \operatorname{curl}\mathbf{u},$$

which becomes the main difficulty.

This approach has been developed in 2D and applied successively to the case of a uniform flow and to the case of a non-vanishing parallel shear flow. In the first case, ψ can be computed a priori [2,8] and in the second case, it is explicitly related to $\mathbf{u}$ by a non-local convolution formula [4]. A simplified approach has been proposed in the case of a low Mach flow [3]: we can then replace the exact non-local expression of ψ by a simple local formula. This low Mach approach has been validated in the case of both a potential and a parallel flow, for which reference solutions are available.

The objective of the present paper is to get rid of the low Mach hypothesis. The main part of the paper is devoted to the theoretical study of the time-harmonic advection equation satisfied by ψ. The well-posedness results that we establish cannot be directly deduced from known results on the classical advection equation [7], but the techniques we use are inspired from [1].

The outline of the paper is the following. The model is briefly described in Section 2, including the augmented equation for $\mathbf{u}$, the hydrodynamic equation for ψ and the Perfectly Matched Layers. Details can be found in [3]. Section 3 is devoted to the theoretical study of the time-harmonic advection equation. Well-posedness is deduced from an inf-sup condition, which is proved for a flow which "fills" the domain, in the sense of [1]. These results are used in Section 4 to prove that the complete problem in $(\mathbf{u},\psi)$ with Perfectly Matched Layers is of Fredholm type if the flow varies slowly. A numerical method, coupling classical finite elements for $\mathbf{u}$ with a Discontinuous Galerkin scheme for ψ is finally described in Section 5 and some numerical results are presented.