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On Pattern Selection in Three-Dimensional Bénard-Marangoni Flows

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Abstract. In this paper we study Bénard-Marangoni convection in confined containers where a thin fluid layer is heated from below. We consider containers with circular, square and hexagonal cross-sections. For Marangoni numbers close to the critical Marangoni number, the flow patterns are dominated by the appearance of the well-known hexagonal convection cells. The main purpose of this computational study is to explore the possible patterns the system may end up in for a given set of parameters. In a series of numerical experiments, the coupled fluid-thermal system is started with a zero initial condition for the velocity and a random initial condition for the temperature. For a given set of parameters we demonstrate that the system can end up in more than one state. For example, the final state of the system may be dominated by a steady convection pattern with a fixed number of cells, however, the same system may occasionally end up in a steady pattern involving a slightly different number of cells, or it may end up in a state where most of the cells are stationary, while one or more cells end up in an oscillatory state. For larger aspect ratio containers, we are also able to reproduce dislocations in the convection pattern, which have also been observed experimentally. It has been conjectured that such imperfections (e.g., a localized star-like pattern) are due to small irregularities in the experimental setup (e.g., the geometry of the container). However, we show, through controlled numerical experiments, that such phenomena may appear under otherwise ideal conditions. By repeating the numerical experiments for the same non-dimensional numbers, using a different random initial condition for the temperature in each case, we are able to get an indication of how rare such events are. Next, we study the effect of symmetrizing the initial conditions. Finally, we study the effect of selected geometry deformations on the resulting convection patterns.

AMS subject classifications: 65C20, 76D05, 76E06, 65N35

Key words: Bénard-Marangoni, spectral elements, pattern formation.

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1 Introduction

We consider Bénard-Marangoni convection in confined containers where a thin fluid layer is heated from below. This problem has previously been studied extensively, both experimentally as well as computationally. An intriguing feature of this problem is the formation of hexagonal convection cells from random initial conditions; see [2,17,22].

It has been found that the onset of convection can originate from two different effects; it can be caused by buoyancy effects due to the fact that the density is a function of the temperature, or it can be due to variations in the surface tension, i.e., thermocapillary forces. Both effects can also be present at the same time. In fact, the concurrent presence of the two effects was a source of confusion for years. Bénard himself had an incorrect interpretation of which effect was the dominant one in his original experiments [2], which lead to Rayleigh's subsequent stability analysis also being done under the assumption of buoyancy-driven convection [34]. It took several decades before this misunderstanding was cleared up; see [8,32]. It has also later been shown [3] that buoyancy forces induce rolls in a shallow layer with a free upper surface when the Marangoni number is zero.

The benchmark experiments in later years have been those of Koschmieder and Prahl [24]. In his monograph [22], Koschmieder gives a comprehensive overview of the problem, both from a theoretical and experimental point of view.

Ramon et al. [33] investigated the pattern formation predicted for small aspect ratio containers. They obtained results that confirmed the predictions made by Rosenblat et al. [35] based on linear stability theory.

Yu et al. [43] studied the pattern formation computationally using a least-squares finite-element-based method. Their focus was on reproducing the experimental results of Koschmieder [22]. They obtained results which were in good agreement, both at the qualitative level, reproducing the patterns from the experiments, as well as predicting the critical Marangoni numbers. Their simulations were started with an initial condition consisting of a superposition of all Fourier modes that were resolved on their grids.

Dauby et al. [14] used a spectral Tau method to determine the critical Marangoni number, as well as the convective pattern at the threshold. The simulations were performed for rectangular containers with rigid walls, with the aspect ratio as the main parameter. The influence of a non-vanishing gravity and a non-zero Biot number at the free surface was examined. The authors showed that the convective pattern above the threshold may differ substantially from the pattern predicted from linear stability theory due to the presence of the rigid walls. In a follow-up study [15] the linear instability in circular containers were investigated. The authors numerically confirmed the principle of "exchange of stabilities".

In a more recent study [30], Medale and Cerisier investigated numerically the convection patterns in containers of various shapes and sizes using a finite volume method. They also found results which were in very good agreement with the experimental results from [24].

Bjøntegaard and Rønquist [7] studied numerically the effects of a deformable free