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Improvement on Spherical Symmetry in Two-Dimensional Cylindrical Coordinates for a Class of Control Volume Lagrangian Schemes

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Abstract. In [14], Maire developed a class of cell-centered Lagrangian schemes for solving Euler equations of compressible gas dynamics in cylindrical coordinates. These schemes use a node-based discretization of the numerical fluxes. The control volume version has several distinguished properties, including the conservation of mass, momentum and total energy and compatibility with the geometric conservation law (GCL). However it also has a limitation in that it cannot preserve spherical symmetry for one-dimensional spherical flow. An alternative is also given to use the first order area-weighted approach which can ensure spherical symmetry, at the price of sacrificing conservation of momentum. In this paper, we apply the methodology proposed in our recent work [8] to the first order control volume scheme of Maire in [14] to obtain the spherical symmetry property. The modified scheme can preserve one-dimensional spherical symmetry in a two-dimensional cylindrical geometry when computed on an equal-angle-zoned initial grid, and meanwhile it maintains its original good properties such as conservation and GCL. Several two-dimensional numerical examples in cylindrical coordinates are presented to demonstrate the good performance of the scheme in terms of symmetry, non-oscillation and robustness properties.

AMS subject classifications: 65M06, 76M20

Key words: Control volume Lagrangian scheme, spherical symmetry preservation, conservative, cell-centered, compressible flow, cylindrical coordinates.

1 Introduction

The Lagrangian method is one of the main numerical methods for simulating multidimensional fluid flow, in which the mesh moves with the local fluid velocity. It is widely

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used in many fields for multi-material flow simulations such as astrophysics, inertial confinement fusion (ICF) and computational fluid dynamics (CFD), due to its distinguished advantage in capturing material interfaces automatically and sharply. There are two kinds of Lagrangian methods. One is built on a staggered discretization in which velocity (momentum) is stored at vertices, while density and internal energy are stored at cell centers. The density/internal energy and velocity are solved on two different control volumes, see, e.g., [1, 3, 18]. This kind of Lagrangian schemes usually uses an artificial viscosity term, for example [4,5,18], to ensure the dissipation of kinetic energy into internal energy through shock waves. The other is based on the cell-centered discretization in which density, momentum and total energy are all centered within cells and evolved on the same control volume, e.g., [6, 7, 10, 13, 15, 17]. This kind of schemes does not require the addition of an explicit artificial viscosity for shock capturing. Numerical diffusion is implicitly contained in the Riemann solvers.

It is a critical issue for a Lagrangian scheme to keep certain symmetry in a coordinate system different from that symmetry. For example, in the simulation of implosions, since the small deviation from spherical symmetry due to numerical errors may be amplified by Rayleigh-Taylor or other instabilities which may lead to unexpected large errors, it is very important for the scheme to keep the spherical symmetry. In the past several decades, many research works have been performed concerning the spherical symmetry preservation in two-dimensional cylindrical coordinates. The most widely used method that keeps spherical symmetry exactly on an equal-angle-zoned grid in cylindrical coordinates is the area-weighted method [2, 3, 14, 20, 22, 23]. In this approach one uses a Cartesian form of the momentum equation in the cylindrical coordinate system, hence integration is performed on area rather than on the true volume in cylindrical coordinates. However, these area-weighted schemes have a flaw in that they may violate momentum conservation. Margolin and Shashkov used a curvilinear grid to construct symmetry-preserving discretizations for Lagrangian gas dynamics [16]. In our recent work [8], we have developed a new cell-centered control volume Lagrangian scheme for solving Euler equations of compressible gas dynamics in two-dimensional cylindrical coordinates. Based on the strategy of local coordinate transform and a careful treatment of the source term in the momentum equation, the scheme is designed to be able to preserve one-dimensional spherical symmetry in a two-dimensional cylindrical geometry when computed on an equal-angle-zoned initial grid. A distinguished feature of our scheme is that it can keep both the symmetry and conservation properties on the straight-line grid. However, our scheme in [8] does not satisfy the geometric conservation law (GCL).

In [14], Maire developed a class of high order cell-centered Lagrangian schemes for solving Euler equations of compressible gas dynamics in cylindrical coordinates. A node-based discretization of the numerical fluxes is given which makes the finite volume scheme compatible with the geometric conservation law. Both the control volume and area-weighted discretizations of the momentum equations are presented in [14]. The control volume scheme is conservative for mass, momentum and total energy, and satisfies a local entropy inequality in its first-order semi-discrete form. However, it does not