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TE Mode Mixing Dynamics in Curved Multimode Optical Waveguides

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Abstract. Propagation of light through curved graded index optical waveguides supporting an arbitrary high number of modes is investigated. The discussion is restricted to optical wave fields which are well confined within the core region and losses through radiation are neglected. Using coupled mode theory formalism, two new forms for the propagation kernel for the transverse electric (TE) wave as it travels along a curved two-dimensional waveguide are presented. One form, involving the notion of "bend" modes, is shown to be attractive from a computational point of view as it allows an efficient numerical evaluation of the optical field for sharply bent waveguides.

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Key words: Curved waveguides, Coupled Mode Theory, harmonic oscillator.

1 Introduction

Large multimode optical fibres are finding increasing application in many areas of applied science. In practice, macrobending occurs in a large deflection of the fibre axis such as that associated with spooling or the presence of loops. These deviations influence the signal propagation as a result of mode coupling phenomena. This, in turn influences the intermodal dispersion that may limit the achievable data transmission rate. Bends can also be imposed and designed so as to convert from fundamental to the high-order modes in optical fibres [1]. Wave propagation in a bent waveguide can be analyzed via various means ranging from numerical techniques such as the popular Beam Propagation Method (see for instance [2] and references therein) to semi-analytical approaches like

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the Ray Tracing technique [3] or the Beam Tracing Method [4]. For certain waveguide for which propagation modes are analytically known or, at least, can be easily numerically calculated, the Coupled Mode Theory (CMT) offers an efficient alternative for describing the propagation along curved waveguides [3, 5]. Basically, the CMT transforms the original wave equation into a system of N first order ordinary differential equations (N is the number of modes) also called coupling matrix which is then solved using standard integration schemes. Problems arise when the number of propagation modes becomes too large: plastic optical fibres, for instance, can support several hundred thousand up to a million of modes. In these extreme cases, standard coupled mode theory becomes numerically intractable because of the computational overhead, and so there is a need for devising new strategies. The aim of this work is to shed a new light on this issue and to propose an improvement of the standard CMT by considering the coupling of the so-called "bend" modes. Bend modes are local eigenmodes that satisfy the wave equation in the curved waveguide with *constant* curvature. By construction, these modes are decoupled for circular bends and propagate almost adiabatically if the radius of curvature changes sufficiently slowly. More generally, the associated coupling matrix is nearly diagonal in most cases whereas the use of standard CMT would yield a fully populated matrix.

In this paper, the discussion is restricted to slowly varying planar waveguides with a parabolic graded index profile. Furthermore, the optical wave field is assumed to be well confined within the core region, and losses through radiation are neglected. It is further assumed that the waveguide is weakly guiding and that the paraxial approximation holds. Under these assumptions, the problem is shown to be equivalent to the classical time-independent harmonic oscillator. In this scenario, the solution admits an integral formulation involving the Feynman propagator; this is discussed in Section 2 and Section 3. Using the coupled mode theory (CMT) formalism, it is shown that the coupling matrix can be integrated analytically and the exact solution is recovered numerically via the computation of a matrix exponential. The theory is presented for both standard and improved CMT in Section 4 and Section 5. Numerical experiments carried out in Section 6 confirm the efficacy and advantages of the proposed approach.

2 Problem statement

We aim to study the propagation of a monochromatic TE (transverse electric) wave $E = \hat{E}e^{-i\omega t}$ in a weakly guiding two-dimensional dielectric waveguide whose graded-index profile n in the core of width $2a$ has the parabolic form

$$n^2(u) = n_0^2\left(1-2\Delta\left(\frac{u}{a}\right)^2\right), \qquad |u| \le a, \tag{2.1}$$

where $\Delta = (n_0^2 - n_c^2)/2n_0^2$ denotes the usual profile height parameter, n_0 is the refractive index along the waveguide's axis and n_c is the index of the cladding. Eq. (2.1) is naturally