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Simultaneous Scatterer Shape Estimation and Partial Aperture Far-Field Pattern Denoising

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Abstract. We study the inverse problem of recovering the scatterer shape from the far-field pattern(FFP) in the presence of noise. Furthermore, only a discrete partial aperture is usually known. This problem is ill-posed and is frequently addressed using regularization. Instead, we propose to use a direct approach denoising the FFP using a filtering technique. The effectiveness of the technique is studied on a scatterer with the shape of the ellipse with a tower. The forward scattering problem is solved using the finite element method (FEM). The numerical FFP is additionally corrupted by Gaussian noise. The shape parameters are found based on a least-square error estimator. If $\widetilde{u}_\infty$ is a perturbation of the FFP then we attempt to find Γ, the scatterer shape, which minimizes $\| u_\infty - \widetilde{u}_\infty \|$ using the conjugate gradient method for the denoised FFP.

AMS subject classifications: 81U40

Key words: Scattering inverse problem, far field pattern.

1 Introduction

We consider the inverse problem of recovering the scatterer shape from the FFP of the scattered wave. Inverse problems of this type occur in various application such as remote sensing, ultrasound tomography, seismic imaging and radar/sonar detection. They are difficult to solve since they are ill-posed and nonlinear. The ill-posedness is frequently addressed via regularization. Many of the reconstruction methods, such as linear sampling, factorization [4], incorporate some type of regularization.

We propose to directly smooth the (noisy) FFP before the reconstruction. After smoothing, any reconstruction method can be used. For difficult problems, especially with many free parameters, one will frequently require a regularization in addition to the smoothing.

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However, for the simpler problems presented here no regularization is used. Our technique may be interpreted as an implicit regularization imposed on the FFP rather than on the solution. The efficiency of the proposed technique is demonstrated on the problem of recovering the parameters of the scatterer with the shape of the ellipse with a tower (hard case). This shape may be considered as a simplified imitation of a real submarine. There are three parameters describing the shape in this case: the ellipse semi-axis a and b and the height of the tower h (see Fig. 1).

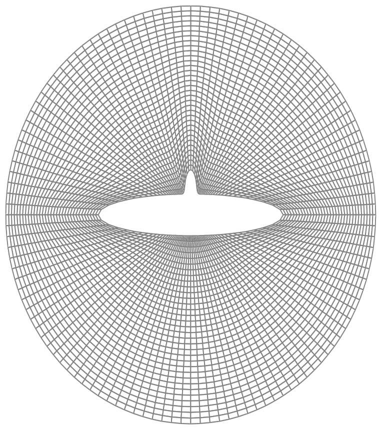

Figure 1: Ellipse with the tower.

2 General description of the problem

The problem is formulated as recovering a body shape Γ from the measured FFP. Usually, the body shape Γ is parameterized by a few parameters that should be recovered. The noise source may be either experimental or computational. The problem is complicated by the fact that in practice only a fraction of the data can be obtained. The given FFP is only measured at discrete angles and one can not usually obtain the data in a complete circle/sphere in the far field but only in some portion of the FFP.

We consider the problem for the total wave $u=u_{inc}+u_s$ in terms of the incoming wave, $u_{inc}=e^{ikx\cdot d}, |d|=1$, where d is the direction of the incident plane wave. We solve the reduced wave equation in two dimensions exterior to a given body, so

$$\Delta^2 u+k^2 u=0, \quad \text{in } \mathbb{R}^2\backslash\overline{\Omega},$$

where Ω is the scattering object. Along the boundary one can impose either a Dirichlet condition corresponding to a sound-soft body or a Neumann condition corresponding to a sound-hard body. In this study we consider an impenetrable sound-hard body and so