

## Transparent Boundary Conditions for Elastic Anisotropic VTI Media: Axially Symmetric Case

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**Abstract.** Transparent boundary conditions (TBCs) for anisotropic vertical transverse isotropic VTI medium are formulated for the axially symmetric case. The high accuracy of the derived TBCs and their long-time stability are demonstrated in numerical experiments. The TBCs are represented in terms of the vertical component of the velocity vector and tangential component of the stress tensor that facilitates the easy implementation of the boundary condition into the finite-difference staggered-grid scheme.

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### 1 Introduction

Numerical modeling of wave propagation in an unbounded physical domain is usually performed in a finite computational domain with nonreflecting boundary conditions applied on its boundary; these conditions should guarantee low (ideally no) spurious reflected waves at the boundary. In the context of acoustic, elastic, and electromagnetic wave propagation, many publications could be mentioned, but we refer just to the reviews [4, 11], see also the references therein.

Formulation of accurate, stable, and computationally efficient nonreflecting conditions in the case of elastic *anisotropic* media has become a hot topic of research in recent years, basically after Becache et al. [2] demonstrated that although perfectly matched layer (PML) [3] is good for isotropic media, it may be unstable for anisotropic media. Several interesting approaches for anisotropic elastic problems, such as a multiaxial PML [6] and truncation method on a base of optimal grids [5], have been proposed just recently. Here, we address the problem of constructing nonreflecting boundary conditions for anisotropic elastic media in the framework of transparent boundary conditions, [1, 7, 8].

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For isotropic media, transparent boundary conditions (TBCs) have been obtained for many problems, including acoustic, linearized Euler, and Maxwell equations. Typically TBCs are described by an integrodifferential operator represented by a sum of local terms, like time and spatial derivatives, and a nonlocal term (an integral of convolutional type). In simple cases, e.g. a wave equation in homogeneous media and the boundary of a simple shape (planar, spherical, and cylindrical), either the convolutional kernel itself or its Laplace transform is obtained analytically. The alternative for the cases when the analytical formulas are unknown is the recently proposed quasi-analytical TBCs [9], where the Laplace transform of the kernel is calculated numerically for anisotropic vertically transverse isotropic (VTI) media for the axially symmetric case,  $(r,z)$ -geometry. However media with smooth  $z$ -dependent parameters in the exterior domain are allowed in the approach [9] and it is very computationally expensive.

In this work we obtain analytical formulas of TBC for the particular, but practically important case of axially symmetric anisotropic elastic VTI media. The derivation of TBC for this case is a more technically sophisticated task than that of TBCs for an acoustic scalar equation [1]. Fortunately, in spite of complicated intermediate formulas, the final form of TBC is relatively simple. We restrict here to the case of a cylindrical boundary and homogeneous media in the exterior domain, though any arbitrary complex medium is allowed in the interior domain. Similar to other cases (excluding the 1D wave equation), TBC is described with the operator that contains both local and non-local terms in time and space. The local term can be used for the problems that require moderate accuracy. To achieve higher accuracy we should include the non-local term, which is approximated via the convolution with a sum of exponentials according to [1, 8] for the efficient numerical implementation.

The rest of the paper is organized as follows. We start with the problem formulation in Section 2. Section 3 is devoted to the derivation of analytical formulas for TBC. Approximation of the boundary condition is discussed in Section 4, and discretization for a finite-difference staggered-grid scheme is outlined briefly in Section 5. The numerical experiments and their results are described in Section 6.

## 2 Problem formulation

We consider a stress-velocity formulation of the elastodynamics equations for the rotationally symmetric case in anisotropic VTI media where the equations of motion are

$$\rho \frac{\partial u_r}{\partial t} = \frac{\partial \sigma_{rr}}{\partial r} + \frac{\sigma_{rr}}{r} - \frac{\sigma_{\varphi\varphi}}{r} + \frac{\partial \sigma_{rz}}{\partial z} + \rho S_r, \quad (2.1a)$$

$$\rho \frac{\partial u_z}{\partial t} = \frac{\partial \sigma_{rz}}{\partial r} + \frac{\sigma_{rz}}{r} + \frac{\partial \sigma_{zz}}{\partial z} + \rho S_z, \quad (2.1b)$$