Commun. Comput. Phys.
doi: 10.4208/cicp.281209.150610s

Vol. **11**, No. 2, pp. 647-659
February 2012

Full Aperture Reconstruction of the Acoustic Far-Field Pattern from Few Measurements

Hélène Barucq[1], Chokri Bekkey[2] and Rabia Djellouli[3,*]

[1] *INRIA Bordeaux Sud-Ouest Research Center, Team Project Magique-3D, & LMA/CNRS UMR 5142, Université de Pau et des Pays de l'Adour, France.*
[2] *Faculté des Sciences de Monastir, Tunisia.*
[3] *Department of Mathematics, California State University Northridge & Interdisciplinary Research Institute for the Sciences, IRIS, USA.*

Received 28 December 2009; Accepted (in revised version) 15 June 2010

Available online 24 October 2011

Abstract. We propose a numerical procedure to extend to full aperture the acoustic far-field pattern (FFP) when measured in only few observation angles. The reconstruction procedure is a multi-step technique that combines a *total variation* regularized iterative method with the standard Tikhonov regularized pseudo-inversion. The proposed approach distinguishes itself from existing solution methodologies by using an exact representation of the total variation which is crucial for the stability and robustness of Newton algorithms. We present numerical results in the case of two-dimensional acoustic scattering problems to illustrate the potential of the proposed procedure for reconstructing the full aperture of the FFP from very few noisy data such as *backscattering* synthetic measurements.

AMS subject classifications: 35J05, 78A46, 74J20, 65F22, 65T40, 35N05

Key words: Acoustic scattering problem, limited aperture, inverse obstacle problem, ill-posed problem, total variation, Tikhonov regularization, Newton method.

1 Introduction

The determination of the shape of an obstacle from the knowledge of some scattered far-field patterns (FFP), and assuming some a priori knowledge about the characteristics of the surface of the obstacle is one of the most basic problems arising in the inverse scattering field. However, this inverse obstacle problem (IOP) is very difficult to solve from both mathematical and numerical view points due to its ill-posed nature and — to some extent — to its nonlinearity. The numerical determination of the obstacle becomes more

*Corresponding author. *Email addresses:* `helene.barucq@univ-pau.fr` (H. Barucq), `chokri.bekkey@ipeit.rnu.tn` (C. Bekkey), `rabia.djellouli@csun.edu` (R. Djellouli)

 ©2012 Global-Science Press

challenging when the FFP is not measured entirely around the obstacle but only in a limited sector (limited aperture), which is the case in most applications. Nevertheless, and because of its importance to many applications such as sonar, radar, geophysical exploration, medical imaging and nondestructive testing [1], applied mathematicians and engineers devoted an important effort and attention, in the last three decades, to the design of solution methodologies for solving numerically IOP problems (see for example the overview in [4] and references therein). The numerical results reported in the literature indicate that the success in the reconstruction of the sought-after shape of an obstacle depends strongly on the *quality* of the given FFP measurements: the aperture (range of observation angles) and the level of noise in the data (accuracy of measurements). In particular, there is no hope — at least by the current numerical methods — to solve the IOP when the FFP is measured in small apertures (less than $\pi/4$) even if the data are noise free (see for example [5–8, 10] among others). Consequently, the development of numerical procedures to enrich (increase the size) the set of FFP measurements when given in a small aperture could become a key step for solving efficiently IOP problems. Note that, from a mathematical point of view, it is always possible to extend the FFP *uniquely* to the entire circle S when given in a (continuous) subset of S. This unique determination is due to the analyticity of the FFP [2]. However, the numerical extension from the knowledge of a (discrete) subset of the FFP is a very challenging problem. Indeed, such extension can be formulated as an inverse problem that is *extremely* ill-posed due to the analyticity of the FFP. Therefore, any numerical procedure for extending (enriching) the FFP measurements must address efficiently the ill-posed nature of this inverse problem.

Previous attempts to solve the inverse problem characterizing the extension of the FFP were based on standard L^2-Tikhonov regularization techniques on the FFP field [9] as well on its first and second derivative [10]. The extension was (to some extent) successful *only* when the range of measurements is given in an aperture larger than $\pi/2$. These procedures fail to address situations of practical interest, that is when only one measurement (backscattering) or very few measurements are available. Recently, the authors suggested a multistep computational procedure that extends few measurements, such as backscattering data, to *full* aperture (360^o-aperture) [11]. The proposed procedure employs, in its first two steps, a regularized Newton-type algorithm where the *total variation* (TV) of the FFP is incorporated to restore the stability to the inverse problem. The total variation of the FFP, which can be viewed as the L^1-norm of the first derivative of the FFP, is evaluated at each Newton iteration approximatively using a finite difference scheme (see Section 3.2 and the appendix in [11]). The first two-steps of the procedure allow to extend the FFP to at least a $\pi/2$-aperture. The third step of the proposed procedure uses a Tikhonov-type regularization technique that is known to be efficient when the data are given on an aperture larger than $\pi/2$. The results delivered by this procedure are very promising, especially in the presence of low levels of noise in the data. We propose in this paper to modify this reconstruction strategy by employing an *exact* representation of the total variation of the FFP. The use of such an expression is expected to retain more stability and robustness that are needed for the convergence of the Newton algorithm