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Numerical Simulation of Free Surface by an Area-Preserving Level Set Method

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Abstract. We apply in this study an area preserving level set method to simulate gas/water interface flow. For the sake of accuracy, the spatial derivative terms in the equations of motion for an incompressible fluid flow are approximated by the fifth-order accurate upwinding combined compact difference (UCCD) scheme. This scheme development employs two coupled equations to calculate the first- and second-order derivative terms in the momentum equations. For accurately predicting the level set value, the interface tracking scheme is also developed to minimize phase error of the first-order derivative term shown in the pure advection equation. For the purpose of retaining the long-term accurate Hamiltonian in the advection equation for the level set function, the time derivative term is discretized by the sixth-order accurate symplectic Runge-Kutta scheme. Also, to keep as a distance function for ensuring the front having a finite thickness for all time, the re-initialization equation is used. For the verification of the optimized UCCD scheme for the pure advection equation, two benchmark problems have been chosen to investigate in this study. The level set method with excellent area conservation property proposed for capturing the interface in incompressible fluid flows is also verified by solving the dam-break, Rayleigh-Taylor instability, two-bubble rising in water, and droplet falling problems.

AMS subject classifications: 65E05, 76T10

Key words: Level set method, phase error, upwinding combined compact scheme, Hamiltonian, symplectic Runge-Kutta, area conservation property.

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1 Introduction

Along the interface between different phases, surface tension becomes essential in affecting manufacture processes of etching, deposition, lithography, image processing and crystal growth [1]. Studies of these complex phenomena in air-water interfacial dynamics, breaking surface wave, solidification-melt dynamics, two-phase reacting flow, and flow-structure interaction involve getting the temporal surface advancement driven by the carrier flow at different physical loadings. The necessity of capturing or tracking the time-evolving interface with possible sharply-varying surface topology and front propagation speed can make the simulation of differential equations governing their respective two-phase fluid flows an even challenging topic.

Several popular methods such as the vortex method [2], boundary integral method [3], volume of fluid (VOF) method [4], front tracking method [5], and phase field method [6–8] have been successfully applied to predict the air/water interface. One can also combine the level set and projection methods to avoid explicitly tracking the interface [9]. There exist some advantages and disadvantages of applying these interface capturing methods and one is difficult to distinguish which method is superior to the others. ALE (Arbitrary Lagrangian Eulerian) [10] and MAC (Marker and Cell) [11] methods are the two commonly applied interface tracking methods which have been known to be very efficient in modeling a small interface deformation. For an interface undergoing a large deformation, a fairly expensive re-meshing procedure is needed. As the surface tension needs to be considered in the simulation of incompressible two-phase flows, the volume of fluid and level set methods turn out to be more commonly referred to. The VOF method, which is normally represented by a color function, has the ability of conserving the volume of each fluid phase more exactly.

After the pioneering work of Osher and Sethian [12], progress towards refining the level set formulations has made this method a good candidate to simulate the flow problems that involve moving interfaces. Given a smooth level set function for the interface, both interface and its curvature can be easily transported and accurately calculated, respectively. Choice of a proper signed distance function for the sake of re-shaping level set function and implementing a re-initialization procedure for the purpose of enhancing numerical stability are normally required in the level set methods [13]. There is no guarantee that re-initialization process can preserve flow volume or area in time. In each time step, fluid mass of a small quantity may, therefore, be lost or gained. To overcome the problem resulting from the application of level set methods, the particle level set method [14], level set method with mass correction procedure [15], coupled level set and volume of fluid method [16], and adaptive tree method [17] have been proposed. One can refer to the excellent review books of Osher and Fedkiw [18] and Sethian [19] for the details of level set methods. Conservative level set method with the built-in conservative property will be developed in the current incompressible flow simulation to preserve fluid mass [20, 21].

In this paper, we present an upwinding combined compact difference (UCCD) scheme