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Optimal L^2 Error Estimates for the Interior Penalty DG Method for Maxwell's Equations in Cold Plasma

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Abstract. In this paper, we consider an interior penalty discontinuous Galerkin (DG) method for the time-dependent Maxwell's equations in cold plasma. In Huang and Li (J. Sci. Comput., 42 (2009), 321–340), for both semi and fully discrete DG schemes, we proved error estimates which are optimal in the energy norm, but sub-optimal in the L^2-norm. Here by filling this gap, we show that these schemes are optimally convergent in the L^2-norm on quasi-uniform tetrahedral meshes if the solution is sufficiently smooth.

AMS subject classifications: 65N30, 35L15, 78-08

Key words: Maxwell's equations, cold plasma, discontinuous Galerkin method.

1 Introduction

Recently, there is a growing interest in the finite element modeling and analysis of Maxwell's equations (see books [7, 14, 21] and references cited therein). However, most work are still limited to the simple medium (such as vacuum) case. On the other hand, dispersive media (whose physical parameters are wavelength dependent) are ubiquitous. Examples include human tissue, soil, snow, ice, plasma, optical fibers and radar-absorbing materials. Hence the study of how electromagnetic waves interacting with dispersive media becomes an important subject.

Though the original discontinuous Galerkin (DG) method has been known since its introduction in 1973 by Reed and Hill, it was only recently that DG regained its popularity in solving various differential equations. It is known that the DG method offers great flexibility in the mesh construction by allowing different types of elements, non-matching grids, and even varying polynomial orders. Due to the imposition of weak continuity across element interfaces, the DG method is easy for parallel implementation.

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A detailed overview of the evolution of the DG methods from 1973 to 1999 is provided by Cockburn et al. [6]. More details and early references on DG can be found in [2,6].

Some DG methods have been developed for Maxwell's equations in the simple medium case [4, 5, 8, 9, 13, 15, 22] in the past decade. We like to remark that most of the DG methods are based on writing the Maxwell's equations in first-order hyperbolic systems; while [9,15] treated the Maxwell's equations in second order vector wave equation. Some most recent developments of DG methods for wave problems can be found in the Proceedings of Waves 2009 [3]. However, the study of DG method for Maxwell's equations in dispersive media is quite limited. In 2004, a time-domain DG method was investigated in [20] for solving the first-order Maxwell's equations in dispersive media. In 2009, we [16] initiated the analysis of the interior penalty DG method for Maxwell's equations in dispersive media. However, the error estimates obtained there is optimal in the energy norm, but sub-optimal in the L^2-norm. In this paper, by borrowing many ideas from [9,11,12,15] originally developed for the curl-curl operator, we manage to prove the optimal error estimates in the L^2-norm for both semi and fully discrete schemes. Note that our proof is slightly different from [9,11,12,15] by considering that our problem is a differential-integral equation instead of the standard vector wave equation. For simplicity, we only consider the cold plasma model here, since analysis of other dispersive media models [17] can be carried out similarly.

By introducing $c_v = (\sqrt{\epsilon_0\mu_0})^{-1}$ as the speed of wave propagation in vacuum, we can rewrite the governing equation for the isotropic nonmagnetized cold electron plasma model [16, Eq. (1)] as

$$\boldsymbol{E}_{tt}+\nabla\times\left(c_v^2\nabla\times\boldsymbol{E}\right)+\omega_p^2\boldsymbol{E}-\boldsymbol{J}(\boldsymbol{E})=0, \quad \text{in } \Omega\times I, \tag{1.1}$$

where $\boldsymbol{E}$ is the electric field, ω_p is the plasma frequency, and $\boldsymbol{J}$ is the polarization current density represented as

$$\boldsymbol{J}(\boldsymbol{x},t;\boldsymbol{E})\equiv\boldsymbol{J}(\boldsymbol{E})=\nu\omega_p^2\int_0^t e^{-\nu(t-s)}\boldsymbol{E}(\boldsymbol{x},s)ds, \tag{1.2}$$

here $\nu\geq 0$ is the electron-neutral collision frequency. In c_v, ϵ_0 and μ_0 represent the permittivity and permeability in vacuum, respectively. Here $I=(0,T)$ is a finite time interval and Ω is a bounded Lipschitz polyhedron in R^3.

Moreover, we assume that the boundary of Ω is a perfect conductor so that

$$\boldsymbol{n}\times\boldsymbol{E}=0, \quad \text{on } \partial\Omega\times I, \tag{1.3}$$

where $\boldsymbol{n}$ denotes the unit outward normal of $\partial\Omega$. Furthermore, we assume that the initial conditions for (1.1) are given as

$$\boldsymbol{E}(\boldsymbol{x},0)=\boldsymbol{E}_0(\boldsymbol{x}) \quad \text{and} \quad \boldsymbol{E}_t(\boldsymbol{x},0)=\boldsymbol{E}_1(\boldsymbol{x}), \tag{1.4}$$

where $\boldsymbol{E}_0(\boldsymbol{x})$ and $\boldsymbol{E}_1(\boldsymbol{x})$ are some given functions.