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Analysis of Convolution Quadrature Applied to the Time-Domain Electric Field Integral Equation

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Abstract. We show how to apply convolution quadrature (CQ) to approximate the time domain electric field integral equation (EFIE) for electromagnetic scattering. By a suitable choice of CQ, we prove that the method is unconditionally stable and has the optimal order of convergence. Surprisingly, the resulting semi discrete EFIE is dispersive and dissipative, and we analyze this phenomena. Finally, we present numerical results supporting and extending our convergence analysis.

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1 Introduction

The exterior electromagnetic scattering problem is often solved in the frequency domain, either by integral equations or by a finite element method. However, if the incoming wave is broad band, it may be attractive to solve the problem in the time domain. In this case we can again choose between volume based methods including the finite difference or discontinuous Galerkin methods and time domain integral equations. It is the latter technique that is the subject of this paper.

Historically the main difficulty with the time domain integral equation (TDIE) approach is stability. In recent years this problem has been overcome by using a time domain Petrov-Galerkin method [22] and this method is now the method of choice [1, 17]. However, in order to maintain stability, it is necessary to perform accurate integrations on

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complex domains obtained by intersecting regions between light cones with triangles in the spatial mesh [21]. This rules out simple quadrature on the spatial mesh, and implies that curvilinear patches (necessary for high order boundary representation) are difficult to implement because of the potentially much more complex domain of integration. In addition it is not easy to account for dispersive and dissipative media in such schemes because it is necessary to have an expression for the fundamental solution (although several engineering approaches have been suggested for specific media [14]). Convolution Quadrature (CQ) offers a potential alternative that is the subject of this paper.

We now detail the problem to be solved. Suppose a perfect conductor occupies a bounded Lipschitz polyhedron $\Omega\subset\mathbb{R}^3$ with boundary $\Gamma:=\partial\Omega$. Let $\Omega^e:=\mathbb{R}^3\backslash\overline{\Omega}$. In addition suppose Γ is connected and simply connected. The time domain electromagnetic scattering problem is then to find $\mathscr{E}:=\mathscr{E}(\boldsymbol{x},t)\in\mathbf{H}(\mathrm{curl},\Omega^e)$ such that

$$\begin{aligned}\mathscr{E}_{tt}+\mathrm{curl}\,\mathrm{curl}\mathscr{E}=0,&\quad \text{in}\,\Omega^e\times[0,T],\\ \mathscr{E}\times\boldsymbol{n}=\boldsymbol{g},&\quad \text{on}\,\Gamma\times[0,T],\\ \mathscr{E}(\cdot,0)=\mathscr{E}_t(\cdot,0)=0,&\quad \text{on}\,\Omega^e.\end{aligned}\tag{1.1}$$

Here, $\boldsymbol{n}$ is the unit outward normal to Γ and $\boldsymbol{g}$ is a given tangential vector field on Γ, vanishing for $t\le 0$, usually obtained as a suitable trace of the incident electromagnetic field (that is $\boldsymbol{g}=-\mathscr{E}^i\times\boldsymbol{n}$ where $\mathscr{E}^i$ is the incident field taken to be a regular solution of Maxwell's equations). For convenience and without loss of generality, we have set the speed of light $c=1$.

To formulate an integral equation for this problem we can use the ansatz that there is a surface tangential field $\boldsymbol{J}:=\boldsymbol{J}(\boldsymbol{x},t)$ such that, for $\boldsymbol{x}\in\Omega^e$,

$$\begin{aligned}&\mathscr{E}_t(\mathbf{x},t)=M(\partial_t)\mathbf{J}(\mathbf{x},t)\\ &\quad:=\int_0^t\int_\Gamma k(\mathbf{x}-\mathbf{y},t-\tau)\mathbf{J}_{tt}(\mathbf{y},\tau)d\sigma_y d\tau-\mathrm{grad}\int_0^t\int_\Gamma k(\mathbf{x}-\mathbf{y},t-\tau)\mathrm{div}_\Gamma\mathbf{J}(\mathbf{y},\tau)d\sigma_y d\tau,\end{aligned}\tag{1.2}$$

where the time domain fundamental solution in three dimensions is $k(\mathbf{x},t):=\frac{\delta(t-|\mathbf{x}|)}{4\pi|\mathbf{x}|}$ (this is just the inverse Fourier transform of the usual frequency domain fundamental solution [8]). In addition div_Γ is the surface divergence.

If we now define the surface tangential projection $\Pi_T\mathbf{u}:=\mathbf{n}\times(\mathbf{u}\times\mathbf{n})|_\Gamma$, let $\boldsymbol{x}$ approach Γ in (1.2), and use the boundary data from (1.1), we obtain the Electric Field Integral Equation (EFIE). In particular, we need to find $\boldsymbol{J}$ such that, for all $\boldsymbol{x}\in\Gamma$ and $0\le t\le T$,

$$\begin{aligned}&V(\partial_t)\mathbf{J}(\mathbf{x},t)\\ &:=\Pi_T\int_0^t\int_\Gamma k(\mathbf{x}-\mathbf{y},t-\tau)\mathbf{J}_{tt}(\mathbf{y},\tau)d\sigma_y d\tau-\mathrm{grad}_\Gamma\int_0^t\int_\Gamma k(\mathbf{x}-\mathbf{y},t-\tau)\mathrm{div}_\Gamma\mathbf{J}(\mathbf{y},\tau)d\sigma_y d\tau\\ &=\mathbf{n}\times\mathbf{g}_t(\mathbf{x},t),\end{aligned}\tag{1.3}$$

where grad_Γ is the surface gradient. Once $\boldsymbol{J}$ is computed, we can compute the electric field $\mathscr{E}$ for $\mathbf{x}\notin\Gamma$ by integrating (1.2). Clearly these integral equations need to be carefully formulated in appropriate function spaces and we do this in the next section.