Effects of Complex System Topology on the Bak-Sneppen Evolution Model

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Abstract. In this paper, we investigate by numerical simulations the Bak-Sneppen model (BSM) for biological evolution on scale-free networks (SFNs) with various degree exponents γ. We find that the punctuated equilibrium is rather robust with respect to the network topology. Furthermore, we analyze the evolution of the critical average fitness $\langle f \rangle_*$ and the exponent τ of $\langle f \rangle_0$ avalanche as a function of α (i.e., the degree exponent γ). Our observations indicate the dependence of evolutionary dynamics of BSM on the complex biosystem topology.

1 Introduction

The Bak-Sneppen model (BSM) [1], which was proposed by Bak and Sneppen in 1993, can generate the punctuated equilibrium behavior observed in the evolution of many species. In the BSM, random numbers f_i, drawn from a uniform distribution between 0 and 1, $p(f)$, are assigned to each species located on an n-dimensional lattice with periodic boundary conditions. At each time step, the extinct species, i.e., the species with the smallest random number, and all its nearest neighboring species, are assigned new random numbers also chosen from $p(f)$. After a long transient process the system evolves into a self-organized critical state which is statistically stationary, where the density of fitness values in the system is uniform above f_c (the self-organized threshold) and vanishes for $f < f_c$, and the avalanches of mutations occur on all scales.

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Despite the fact that it is an oversimplification of the evolution of real species, the BSM shows some common interesting features observed by paleontologists, such as the punctuated equilibrium, power-law probability distributions of lifetimes of species and sizes of extinction events [2]. Since the BSM was introduced, the model has been paid much attention. For instance, Li and Cai [2,3] studied different hierarchy of avalanches, exact equations and scaling relations for $\langle f \rangle_0$ avalanches in the BSM. On the other hand, many real biological networks, such as the food webs [4,5], the metabolic networks [6,7] and the protein networks [8,9], share some universal characteristics such as the small-world effect and the power-law degree distribution $p(k) \sim k^{-\gamma}$. These features may affect the dynamics of the networks on which the species are placed. Hence, it is important to study the effects of complex biosystems topology on the evolutionary dynamics of BSM. Recently, the BSM has been studied on random networks (RNs) [10], small-world networks (SWNs) [11] and scale-free networks (SFNs) [12,13]. Moreno and Vazquez studied the avalanche size distribution and the activity time behavior at nodes with different connectivity of BSM only on a SFN with $\gamma = 3$ [12]. Lee and Kim studied the dependence of the critical fitness f_c and the avalanche size distribution $P(S)$ on the connectivity property of SFN [13], from the viewpoint of microscopic. Here, we focus on the dependence of the evolutionary dynamics of BSM on the complex system topology, especially the exponent of the connectivity distribution, from the viewpoint of macroscopic.

In order to do this, we study the evolutionary dynamics of BSM on SFNs, which is generated by using the static model [15] instead of the preferential attachment growth algorithm [16], with various degree exponent $\gamma \in (2, \infty)$. Generically, the system reaches a steady state where the average fitness $\langle f \rangle$ approaches a critical value $\langle f \rangle_*$ and all the $\langle f \rangle$s are smaller than the critical average fitness $\langle f \rangle_*$ as time $t \to \infty$. And the $\langle f \rangle_*$ decreases with the tunable parameter α, which is related to the degree exponent γ via $\gamma = (1+\alpha)/\alpha$, increasing. On the other hand, SFNs with the degree exponent $2 < \gamma \leq 3$ are physically different from those with $\gamma > 3$ [14]. Furthermore, we focus on the evolution of $\langle f \rangle_*$ and the exponent τ of the $\langle f \rangle_0$ avalanche size distribution $P(S) \sim S^{-\tau}$ as a function of the parameter α, i.e., the degree exponent γ. We find that the complex network structure plays a crucial role in the evolutionary dynamics of BSM. Furthermore, we analyze $\langle f \rangle_*$ as a function of α on SFNs with various system sizes N, and find that the BSM on SFNs with $2 < \gamma \leq 3$ self-organizes into a stationary state where the critical average fitness $\langle f \rangle_* \to 0.5$ for $N \to \infty$, just as $f_c \to 0$ for $N \to \infty$ found in [12, 13]. Finally, we classify the SFNs into three different categories: random for $\gamma > 4.3$, linear for $3 < \gamma < 4.3$ and physical for $2 < \gamma \leq 3$, from the viewpoint of dynamics.

2 Model

Many real biological systems can be well mapped to complex networks, which are sets of nodes $i = 1, 2, \cdots, N$, connected by a number of $l = 1, 2, \cdots, L$ edges. The network is represented by its adjacency matrix A, where $A_{ij} = 1$, if an edge connects nodes i and j