

Rotational Slip Flow in Coaxial Cylinders by the Finite-Difference Lattice Boltzmann Methods

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Abstract. Recent studies on applications of the lattice Boltzmann method (LBM) and the finite-difference lattice Boltzmann method (FDLBM) to velocity slip simulations are mostly on one-dimensional (1D) problems such as a shear flow between parallel plates. Applications to a 2D problem may raise new issues. The author performed numerical simulations of rotational slip flow in coaxial cylinders as an example of 2D problem. Two types of 2D models were used. The first were multi-speed FDLBM models proposed by the author. The second was a standard LBM, the D2Q9 model. The simulations were performed applying a finite difference scheme to both the models. The study had two objectives. The first was to investigate the accuracies of LBM and FDLBM on applications to rotational slip flow. The second was to obtain an experience on application of the cylindrical coordinate system. The FDLBM model with 8 directions and the D2Q9 model showed an anisotropic flow pattern when the relaxation time constant or the Knudsen number was large. The FDLBM model with 24 directions showed accurate results even at large Knudsen numbers.

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Key words: Finite-difference lattice Boltzmann method, rarefied gas flow, rotational slip flow, cylindrical coordinate.

1 Introduction

A rarefied gas flow is represented properly by the Boltzmann equation. However, the Boltzmann equation is an equation in the phase space: physical space plus velocity space. Burden in computing is enormous. Therefore, in an intermediate flow such as the velocity slip, where both the Navier-Stokes flow and the rarefied gas flow co-exist, the lattice Boltzmann method (LBM) and the finite-difference lattice Boltzmann method (FDLBM) are potentially ideal flow solvers if they can represent the rarefied gas flow properly.

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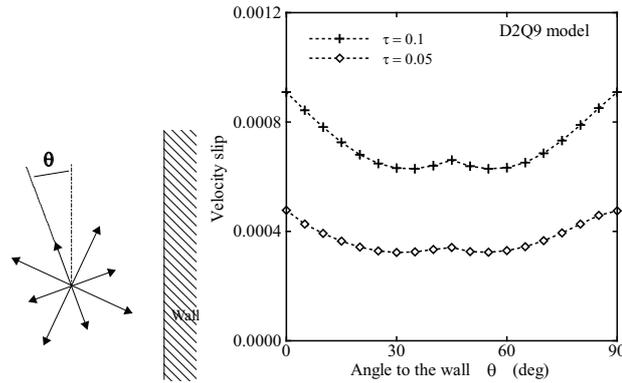


Figure 1: Velocity slip fluctuations by the relative angle to the wall.

There are many recent studies on applications of the LBM and the FDLBM to velocity slip simulations [1–10]. Most studies so far are applications to one-dimensional (1D) problems such as a shear flow between parallel plates. The author conducted a simulation of shear flow between parallel plates using the D2Q9 model [11] and found that the velocity slip fluctuates considerably (32 percents) when the relative angle to the wall changes (see Fig. 1). Therefore, in order to apply the LBM or FDLBM to 2D problems, the dependency on the inclination angle must be decreased by increasing the number of directions of velocity particles.

In this study, the author studied a 2D problem, simulating a rotational slip flow. Two types of 2D models were used. The first are multi-speed thermal FDLBM [12] and its derivative models with different number of velocity particles. The second is the standard LBM model, D2Q9 [13]. The cylindrical coordinate system was adopted in the simulation. To obtain an experience on application of the cylindrical coordinate system to a slip flow is another objective of this study. Before proceeding to the FDLBM simulations, a review of a no-slip solution by the Navier-Stokes analysis and numerical simulations by the continuous Boltzmann equation were conducted to understand the rotational slip phenomena.

Quantities used in this paper are nondimensional based on the reference density ρ_0 , the reference length L , and the reference temperature T_0 (where R is gas constant).

Length (x, y, r)	by L ,
Speed ($u_r, u_\theta, c, c_k, c_{ki\alpha}$)	by $\sqrt{RT_0}$,
Time (t, τ)	by $L/\sqrt{RT_0}$,
Internal energy (e)	by RT_0 ,
Density, distribution function ($\rho, f_{ki}, f_{ki}^{eq}$)	by ρ_0 ,
Distribution function per unit velocity volume (f, f^{eq})	by $\rho_0/(RT_0)$,
Momentum flow ($P_{\theta r}$)	by $\rho_0 RT_0$,