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The Lid-Driven Square Cavity Flow: From Stationary to Time Periodic and Chaotic

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Abstract. Ranging from Re=100 to Re=20,000, several computational experiments are conducted, Re being the Reynolds number. The primary vortex stays put, and the long-term dynamic behavior of the small vortices determines the nature of the solutions. For low Reynolds numbers, the solution is stationary; for moderate Reynolds numbers, it is time periodic. For high Reynolds numbers, the solution is neither stationary nor time periodic: the solution becomes chaotic. Of the small vortices, the merging and the splitting, the appearance and the disappearance, and, sometime, the dragging away from one corner to another and the impeding of the merging—these mark the route to chaos. For high Reynolds numbers, over weak fundamental frequencies appears a very low frequency dominating the spectra—this very low frequency being weaker than clear-cut fundamental frequencies seems an indication that the global attractor has been attained. The global attractor seems reached for Reynolds numbers up to Re = 15,000. This is the lid-driven square cavity flow; the motivations for studying this flow are recalled in the Introduction.

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1 Introduction

The square: the simplest shape—the flow: unexpected and complicated long-term dynamic behavior and the global attractor persisting at extremely large time *t*—this is the lid-driven square cavity flow—an almost fictitious flow [27]—solved many times by various techniques: [1, 10, 19, 21, 23, 26, 35], their results sometime agreeing, sometime disagreeing.

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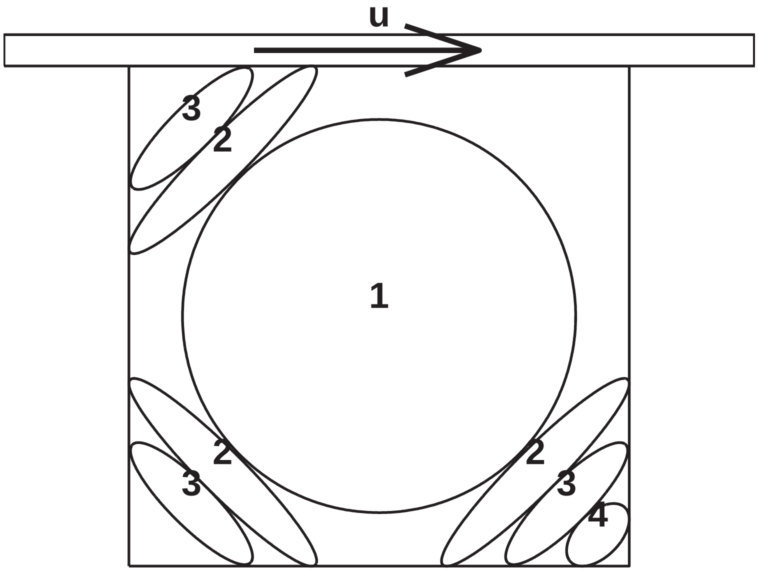

Figure 1: The lid-driven square cavity flow.

The domain is the unit square cavity, and the viscous incompressible flow is governed by the two-dimensional time-dependent incompressible Navier-Stokes equations (NSE) [33] and driven by the upper wall, see Fig. 1. Here, we consider the nondimensionalized NSE in primitive variables with Dirichlet boundary conditions over the domain $\Omega =]0,1[\times]0,1[$; that is

$$\begin{cases} \dfrac{\partial \mathbf{u}}{\partial t} - \nu\Delta\mathbf{u} + c(\mathbf{u},\mathbf{u}) + \nabla p = \mathbf{f} & \text{in } \Omega, \ t>0, \\ \nabla\cdot\mathbf{u} = 0 & \text{in } \Omega, \\ \mathbf{u} = \boldsymbol{\varphi} & \text{on } \Gamma = \partial\Omega, \\ \mathbf{u}(\mathbf{x},0) = \mathbf{u}_0(\mathbf{x}) & \text{in } \Omega, \end{cases} \tag{1.1}$$

where $\mathbf{u}$ is the velocity, p is the pressure, $\nu > 0$ is the kinematic viscosity, $\mathrm{Re} = \nu^{-1}$ is the Reynolds number, $\mathbf{f}$ is the external force, and $c(\mathbf{u},\mathbf{v}) = (\mathbf{u}\cdot\nabla)\mathbf{v}$ represents the convection term. Here, we set $\mathbf{f} = \mathbf{0}$ and consider the boundary conditions

$$\begin{cases} \mathbf{u}(\mathbf{x},\cdot) = (1,0) & \text{if } \mathbf{x} \in \text{upper wall}, \\ \mathbf{u}(\mathbf{x},\cdot) = \mathbf{0} & \text{if } \mathbf{x} \in \text{left, bottom, or right wall}. \end{cases} \tag{1.2}$$

An unexpected balance of viscous and pressure forces makes the fluid to turn into the square cavity. The properties of these forces depending upon the Reynolds number, a hierarchy of vortices develops—the large clockwise-rotating primary vortex (1), whose location occurs toward the geometric center of the square cavity, and several small vortices: the counterclockwise-rotating secondary vortices (2), the clockwise-rotating tertiary