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# Analytical Solution for Waves Propagation in Heterogeneous Acoustic/Porous Media. Part II: The 3D Case

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**Abstract.** We are interested in the modeling of wave propagation in an infinite bilayered acoustic/poroelastic media. We consider the biphasic Biot's model in the poroelastic layer. The first part was devoted to the calculation of analytical solution in two dimensions, thanks to Cagniard de Hoop method. In the first part (Diaz and Ezziani, Commun. Comput. Phys., Vol. 7, pp. 171-194) solution to the two-dimensional problem is considered. In this second part we consider the 3D case.

**AMS subject classifications**: 34B27, 35L05, 35L15, 74F10, 74J05

**Key words**: Biot's model, poroelastic waves, acoustic waves, acoustic/poroelastic coupling, analytical solution, Cagniard-de Hoop's technique, 3D.

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## 1 Introduction

The computation of analytical solutions for wave propagation problems is of high importance for the validation of numerical computational codes or for a better understanding of the reflexion/transmission properties of the media. Cagniard-de Hoop method [1, 2] is a useful tool to obtain such solutions and permits to compute each type of waves (P wave, S wave, head wave$\cdots$) independently. Although it was originally dedicated to the solution of elastodynamic wave propagation, it can be applied to any transient wave propagation problem in stratified media. However, as far as we know, few works have

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been dedicated to the application of this method to poroelastic medium. In [3] the analytical solution of poroelastic wave propagation in an homogeneous 2D medium is provided and in [4] the authors compute the analytical expression of the reflected wave at the interface between an acoustic and a poroelastic layer in two dimension but they do not explicit the expression of the transmitted waves. The coupling between acoustic and poroelastic media is of high interest for the simulation of wave propagation for seismics problem in sea bottom or for ultrasound wave propagation in biological tissues, when the human skin can regarded as a fluid and the bones as a porous medium.

In order to validate computational codes of wave propagation in poroelastic media, we have implemented the codes Gar6more 2D [5] and Gar6more 3D [6] which provide the complete solution (reflected and transmitted waves) of the propagation of wave in stratified 2D or 3D media composed of acoustic/acoustic, acoustic/elastic, acoustic/poroelastic or poroelastic/poroelastic. The 2D code and the 3D code are freely downloadable at

`http://www.spice-rtn.org/library/software/Gar6more2D`

and

`http://www.spice-rtn.org/library/software/Gar6more3D`.

In previous studies [7–9] we have presented the 2D acoustic/poroelastic and poroelastic/poroelastic cases and we focus here on the 3D acoustic/poroelastic case,the 3D poroelastic/poroelastic case will be the object of forthcoming papers.

The paper is organized as follows. We first present the model problem we want to solve and derive the Green problem from it (Section 1). Then we present the analytical solution of wave propagation in a stratified 3D medium composed of an acoustic and a poroelastic layer (Section 2) and we detail the computation of the solution (Section 3). Finally we illustrate our results through numerical applications (Section 4).

## 2 The model problem

We consider an infinite three-dimensional medium ($\Omega = \boldsymbol{R}^3$) composed of an homogeneous acoustic layer $\Omega^+ = \boldsymbol{R}^2 \times ]0,+\infty[$ and an homogeneous poroelastic layer $\Omega^- = \boldsymbol{R}^2 \times ]-\infty,0[$ separated by an horizontal interface $\Gamma$ (see Fig. 1). We first describe the equa-

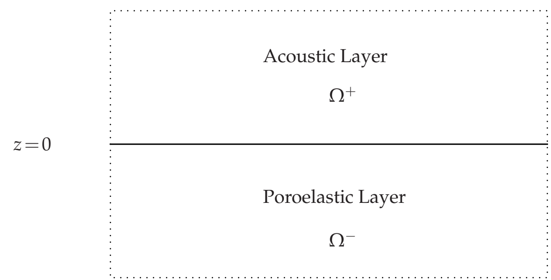


Figure 1: Configuration of the study.