

## Numerical Simulation for a Droplet Fission Process of Electrowetting on Dielectric Device

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**Abstract.** Electrowetting has been proposed as a technique for manipulating droplets surrounded by air or oil. In this paper, we discuss the modeling and simulation of the droplet fission process between two parallel plates inside an electrowetting on dielectric (EWOD) device. Since the gap between the plates is small, we use the two-phase Hele-Shaw flow as a model. While there are several high order methods around, such as the immersed interface methods [1,2], we decide to use two first-order methods for simplicity. A ghost-fluid (GF) method is employed to solve the governing equations and a local level set method is used to track the drop interface. For comparison purposes, the same set of two-phase Hele-Shaw equations are also solved directly using the immersed boundary (IB) method. Numerical results are consistent with experimental observations reported in the literature.

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**Key words:** Electrowetting, ghost fluid method, Hele-Shaw equations, immersed boundary method, local level set method, microfluidics, moving interface, two-phase flow.

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## 1 Introduction

Lab-on-a-chip devices involve miniaturization of many chemical processes onto a single chip. Droplets, as the most common carrier for bio-chemical agents, have been found a growing importance in lab-on-a-chip design. Numerous papers which were centered on droplet operations have been published, c.f. [3–5] and references therein, and droplet-based lab-on-a-chip has been referred to as digital microfluidics. The basic operations include droplet generation; droplet translocation; droplet fusion and droplet fission.

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Amongst the different digital microfluidic systems, electrowetting on dielectric (EWOD) is one of the most promising technique to achieve these goals, because it manipulates discrete droplets rather than a continuous flow.

On the micro-scale, the surface tension forces play a dominate role in the hydrodynamics of a droplet. When a droplet contacts solid electrodes, a wetting force acts on the tri-phase contact line due to electrowetting (changes in the contact angle), and this can be utilized to manipulate the droplet. To avoid electrolysis, an insulating layer is usually inserted between the droplet and electrodes [6–8]. The applications of EWOD devices were discussed extensively in the literature, including microfluid transport [9], tunable optical fiber devices [10], rotating liquid micromotor [11], micro-injection [12], particle separation and concentration control [13]. Other studies focusing on the modeling of EWOD devices can be found in [5,7,14–17].

In this paper we investigate the droplet fission process using a two-phase Hele-Shaw model where the dynamics of both the droplet and the ambient flow is included. We present a ghost fluid (GF) method [19,20] as well as an immersed boundary (IB) method [18] to solve the Hele-Shaw equations. A local level set method [21] is used to track the interface. Our numerical results show that the de-ionized water droplet pinches off without explicit tracking of the interface, contrary to [17] where value of the level set function needs to be artificially reduced to split the droplet.

The rest of the paper is organized as follows. Section 2 explains the basic principle of EWOD and provides a description of the parallel-plate EWOD device and relevant physical parameters. Section 3 presents the Hele-Shaw model for EWOD. Section 4 describes the numerical methods while numerical results are presented in Section 5. Discussion and conclusion are given in Section 6.

## 2 Basic principle of EWOD

It is well known that a droplet on a solid surface spreads or contracts until it has reached the state of minimum free energy, which is determined by cohesive forces in the liquid and the adhesion force between the liquid and the surface. At the tri-phase contact line, the relationship between contact angle  $\theta$  and interfacial tensions is given by Young's equation

$$\cos\theta = \frac{\gamma_{SA} - \gamma_{SL}}{\gamma_{LA}}, \quad (2.1)$$

where  $\gamma_{SA}$  is the solid-ambient fluid surface tension,  $\gamma_{SL}$  is solid-droplet liquid surface tension and  $\gamma_{LA}$  is the droplet liquid-ambient fluid interfacial tension.

When an electric voltage is applied, the change of electric charge distribution at the solid-liquid interface alters the free energy on the surface, inducing a change in wettability of the surface and the contact angle of the droplet [22], which is expressed by the