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Modeling of Long-Range Intra- and Inter-Species Charged Particle Collisions for PIC Simulations

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Abstract. In this paper we present the modelling of elastic intra-species electron-electron and inter-species electron-ion scattering in a plasma on the basis of the Fokker-Planck collision operator. Taking into account the equivalence of this operator with a stochastic differential equation, we propose a Particle-in-Cell based approach for the numerical solution of the Fokker-Planck collision term. As we will see, the introduced numerical concept allows the simulation of the collisional relaxation process in a fully self-consistent fashion.

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Key words: Coulomb collisions in plasmas, collisional relaxation, Fokker-Planck equation, stochastic differential equation, particle-in-cell method.

1 Introduction

A better physical understanding of electrical space propulsion systems like pulsed plasma thrusters [1] as well as a multitude of other discharge driven systems, in general, requires the numerical modelling and simulation of highly rarefied plasma flows. Mathematically, such phenomena demand a kinetic description which is established by the Boltzmann equation. An attractive numerical approach to tackle the non-linear Boltzmann problem consists in a combination of the well-known Particle-in-Cell (PIC) and Monte Carlo

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methods. Basically, the minimal numerical model should accommodate the physics of interaction of charged particles with electromagnetic fields, elastic and inelastic electron-neutral scattering as well as elastic intra- and inter-species charged particle Coulomb collisions. Note, that the endeavor to include plasma particle interactions into fully kinetic simulations, usually carried out with PIC codes, have a long history (for a review see [2]).

In the present paper we focus our attention on elastic intra-species electron-electron and inter-species electron-ion collisions. For the plasmas of interest it is assumed that especially electron-electron interactions play a dominant role. They are crucial in determining the shape of the electron energy distribution function and are responsible for populating the high-energy tail of the velocity distribution to meet finally a Maxwellian distribution function. It is clear from energetic considerations that the high-energy part of the velocity distribution controls reactions like atomic excitation and ionization — the energy sinks for electrons — and to some extent the plasma chemistry. Furthermore, in bounded plasmas high-energy electrons can escape to the wall and thus establish an additional contest between depopulating and population the high-energy tail of the distribution function. Due to the long-range nature of the Coulomb force electron-electron collision is a result of a multiple small-angle scattering process of point charges. Besides inadmissible small time steps which are needed to resolve the individual collision event, electron-electron scattering is not a pure two-body interaction in a plasma because a single electron typically influences many other electrons at the same time. Hence, classical Monte Carlo (MC) tools like hard sphere models for (two-body) short-range reactions [3] seemed to be inadequate. Nevertheless several researchers successfully adapt binary collision models for long-range Coulomb interaction in a plasma, e.g. in a pioneer work Takizuka & Abé [4] proposed a (nonlinear) MC collision operator for PIC models which mimics the Fokker-Planck (FP) operator in Landau form. Later on Ma et al. [5] extended the Takizuka & Abé method for gyrokinetic simulations, where the scattering angle of a binary collision event obeys a Gaussian distribution. For this purpose they suggested a velocity-independent version of the Takizuka & Abé technique which relies on the local thermal velocity. Moreover, these authors introduced a fast and highly efficient implementation scheme for binary collisions. Afterwards, Wang and colleagues [6] improved the Takizuka & Abé ansatz. In particular, they clarified the relation between the newly proposed collision operator and the Landau operator which is equivalent to the FP collision term. Note, that all these authors attempted to show the consistency of their approach with the FP equation. However, none of these methods are derived from the FP collision operator. Definitely noteworthy is the cumulative small-angle scattering approach for the simulation of long-range Coulombs collisions introduced by Nanbu [7]. This approach represents a MC method which was deviated on the basis of physical considerations and do not use any kinetic equation. Although it was shown in [8] that this method is compatible with a first order solution of the Boltzmann equation (in Landau form) in Δt, this cumulative small-angle approach as well as the other mentioned methods are non self-consistent techniques to simulate the evolution of the distribution func-