Commun. Comput. Phys.
doi: 10.4208/cicp.111109.241210s

Vol. **9**, No. 5, pp. 1397-1413
May 2011

Three-Dimensional Lattice Boltzmann Simulation of Two-Phase Flow Containing a Deformable Body with a Viscoelastic Membrane

Toshiro Murayama[1,2], Masato Yoshino[3,4,*] and Tetsuo Hirata[3]

[1] *Department of Mathematics and System Development Engineering, Interdisciplinary Graduate School of Science and Technology, Shinshu University, 4-17-1 Wakasato, Nagano-shi, Nagano 380-8553, Japan.*
[2] *Satellite Venture Business Laboratory (SVBL), Shinshu University, 3-15-1 Tokida, Ueda-shi, Nagano 386-8567, Japan.*
[3] *Department of Mechanical Systems Engineering, Faculty of Engineering, Shinshu University, 4-17-1 Wakasato, Nagano-shi, Nagano 380-8553, Japan.*
[4] *CREST, Japan Science and Technology Agency, 4-1-8 Honcho, Kawaguchi-shi, Saitama 332-0012, Japan.*

Received 11 November 2009; Accepted (in revised version) 24 December 2010

Abstract. The lattice Boltzmann method (LBM) with an elastic model is applied to the simulation of two-phase flows containing a deformable body with a viscoelastic membrane. The numerical method is based on the LBM for incompressible two-phase fluid flows with the same density. The body has an internal fluid covered by a viscoelastic membrane of a finite thickness. An elastic model is introduced to the LBM in order to determine the elastic forces acting on the viscoelastic membrane of the body. In the present method, we take account of changes in surface area of the membrane and in total volume of the body as well as shear deformation of the membrane. By using this method, we calculate two problems, the behavior of an initially spherical body under shear flow and the motion of a body with initially spherical or biconcave discoidal shape in square pipe flow. Calculated deformations of the body (the Taylor shape parameter) for various shear rates are in good agreement with other numerical results. Moreover, tank-treading motion, which is a characteristic motion of viscoelastic bodies in shear flows, is simulated by the present method.

AMS subject classifications: 76D99, 76T99, 74F10, 76M28, 74L15

Key words: Lattice Boltzmann method (LBM), two-phase flow, viscoelastic membrane, shear flow, tank-treading motion.

---

*Corresponding author. *Email addresses:* `s08t206@shinshu-u.ac.jp` (T. Murayama), `masato@shinshu-u.ac.jp` (M. Yoshino), `hirata@shinshu-u.ac.jp` (T. Hirata)

1 Introduction

Problems of solid-fluid two-phase flow containing deformable bodies can be found, for example, in biological fields connected with blood flow in capillaries. In this problem, the interaction between red blood cells (RBCs) and blood plasma becomes important in small blood vessels where the cellular size is comparable to the vessel diameter. A normal RBC is easily deformed, and the deformability of the RBC is related to the erythrocyte configuration, the viscosity of the internal fluid, and the viscoelasticity of the membrane [1,2]. In particular, the elastic behavior of the RBC is determined by the nature of the elastic membrane. Although investigations of the complicated behavior of the RBC are needed, it is difficult to examine the phenomena that are involved, particularly in microscale vessels, by means of experiments. Therefore, numerical simulation is considered to be an effective approach for microscopic investigation of such flow problems.

With regard to numerical studies of solid-fluid two-phase flows, Ramanujan and Pozrikidis [3] studied the deformation of a liquid capsule enclosed by an elastic membrane in shear flows with the boundary element method. Boryczko et al. [4] and Dzwinel et al. [5] have proposed discrete particle models for simulation of RBCs in capillary vessels by the Lagrangian coordinates technique. Tsubota et al. [6] carried out a simulation based on the particle method [7] to examine a peculiar rotary motion (i.e., tank-treading motion [8,9]). Although these studies have produced interesting results, because of the complexity of the algorithms employed for the solid-fluid coupling problems, elaborate computing code is required and computation times are rather long.

Recently, the lattice Boltzmann method (LBM) [10–12] has been developed into an alternative and promising numerical scheme for simulating multicomponent and multiphase fluid flows. In particular, for solid-fluid two-phase flows, Ladd [13–15] was the first to simulate solid-fluid suspensions of spheres in shear flows. In addition, flow problems including deformable bodies are simulated by combining LBM with various methods. Sui et al. [16,17] simulated the motion of a body with elastic membrane in shear flows by using an immersed boundary method. Dupin et al. [18,19] also simulated the motion of red blood cells in fluid flows. They discretized the 3D capsule membrane into flat triangular elements, and the elastic forces acting at the triangle vertices are inserted in LBM nodes. The authors [20] have recently investigated behavior of a deformable body with viscoelastic membranes in two-dimensional flows. In the present study, we extend the numerical method to three-dimensions, and moreover, we take account of changes in surface area of the membrane and in total volume of the body as well as shear deformation of the membrane. By using the method, we simulate the behavior of a viscoelastic body under shear flow and the motion of a viscoelastic body in square pipe flow.

The paper is organized as follows. In Section 2, we describe how to determine the elastic forces acting on the viscoelastic membrane of the body and how to introduce elastic force into the two-phase LBM. In Section 3, we present numerical results of the behavior of a deformable body in two flow fields: under shear flow and in square pipe flow. Finally, concluding remarks are given in Section 4.