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A Discrete Flux Scheme for Aerodynamic and Hydrodynamic Flows

S. C. Fu[1,*], R. M. C. So[1,2] and W. W. F. Leung[1,3]

[1] *Mechanical Engineering Department, Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong.*
[2] *School of Mechanical Engineering, Purdue University, West Lafayette, IN 47907-2088, USA.*
[3] *Research Institute of Innovative Products & Technologies, Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong.*

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Abstract. The objective of this paper is to seek an alternative to the numerical simulation of the Navier-Stokes equations by a method similar to solving the BGK-type modeled lattice Boltzmann equation. The proposed method is valid for both gas and liquid flows. A discrete flux scheme (DFS) is used to derive the governing equations for two distribution functions; one for mass and another for thermal energy. These equations are derived by considering an infinitesimally small control volume with a velocity lattice representation for the distribution functions. The zero-order moment equation of the mass distribution function is used to recover the continuity equation, while the first-order moment equation recovers the linear momentum equation. The recovered equations are correct to the first order of the Knudsen number (Kn); thus, satisfying the continuum assumption. Similarly, the zero-order moment equation of the thermal energy distribution function is used to recover the thermal energy equation. For aerodynamic flows, it is shown that the finite difference solution of the DFS is equivalent to solving the lattice Boltzmann equation (LBE) with a BGK-type model and a specified equation of state. Thus formulated, the DFS can be used to simulate a variety of aerodynamic and hydrodynamic flows. Examples of classical aeroacoustics, compressible flow with shocks, incompressible isothermal and non-isothermal Couette flows, stratified flow in a cavity, and double diffusive flow inside a rectangle are used to demonstrate the validity and extent of the DFS. Very good to excellent agreement with known analytical and/or numerical solutions is obtained; thus lending evidence to the DFS approach as an alternative to solving the Navier-Stokes equations for fluid flow simulations.

AMS subject classifications: 76M25, 76D05
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[*] Corresponding author. *Email addresses:* `mm.scfu@polyu.edu.hk` (S. C. Fu), `mmmcso@polyu.edu.hk` (R. M. C. So), `mmwleung@polyu.edu.hk` (W. W. F. Leung)

1 Introduction

The Bhatnagar-Gross-Krook (BGK)-type [1] modeled lattice Boltzmann equation proves to be a viable alternative to the Navier-Stokes (NS) equations for fluid flow simulations and the associated transport phenomena [2–5]. The original Boltzmann equation was derived by stipulating the assumption of ideal gas and the neglect of the intermolecular interaction. The BGK-type modeled Boltzmann equation (MBE) is a scalar equation governing the transport of a particle distribution function f. In order to solve this equation, a conventional approach is to assume a velocity lattice model, thus giving rise to the lattice Boltzmann method (LBM). The scalar equation for f is then transformed into N number of lattice equations; therefore, the process of solving the vector and tensor NS equations is reduced to using LBM to solve the set of scalar equations. With the development of more and more accurate numerical methods, the BGK-type modeled lattice Boltzmann equation (LBE) found success not only in aerodynamic flows but also in the simulations of thermodynamic flows [6,7]. Extension to one-step aeroacoustics simulations [8,9], to acoustic scattering simulations [10,11], and to shock capturing and shock structure simulations [12,13] has also been successfully demonstrated. The simulations are in good agreement with analytical and other known numerical results.

Most of these studies were focused on gas flows where an ideal gas equation of state was specified. For non-ideal gas and fluid flows with multiple phases and components, the appropriateness of the traditional LBM is doubtful. Since the mean-field approximation is widely used in liquid theory [14], the same approach has been extended to treat non-ideal gas flows, fluid flows with phase transitions and binary immiscible fluids [15–18]. These studies were mainly focused on the recovery of the NS equations. A recent theoretical study to represent hydrodynamic systems through a systematic discretization of the Boltzmann kinetic equation has been attempted [19]; it manages to show an alternative way to recover fluid dynamic equations, from the NS equations to Burnett fluids and beyond. As a result, a systematic approach to derive the NS equations based on the kinetic level of representation is available; the approach is not subject to the assumption of an ideal gas law and the neglect of intermolecular interaction. In principle, this latest approach can be used to treat incompressible flow of liquids, such as the double diffusive phenomenon found in ocean layers [20–22]. However, the application of LBM to this branch of fluid dynamics is still lacking. The present work attempts to examine the validity and extent of the application of LBM-type technique or its variant to oceanography.

Using the Hermite expansion approach, Shan et al. [19] showed that fluid flows can be systematically approximated by constructing higher-order LBE models. Instead of adopting this approach to derive a LBE that could recover the NS equations exactly for incompressible (with constant density assumed) and compressible fluid flows, an alternative approach is sought in the present study. Due to the simplicity of the BGK-type modeled LBE, its extension to hydrodynamic problems is both appropriate and desirable, if the assumption of an ideal gas law can be lifted. The focus is on the recovery of