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Implicit Discontinuous Galerkin Method for RANS Simulation Utilizing Pointwise Relaxation Algorithm

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Abstract. An efficient implicit procedure for the Discontinuous Galerkin (DG) method is developed utilizing a pointwise relaxation algorithm. In the pointwise relaxation, those contributions from the degrees of freedom in own computational cell are accounted for in the implicit matrix inversion. The resulting scheme is shown to be stable with very large CFL numbers for both the Euler and the Navier-Stokes equations for typical test problems. In order to achieve a faster convergence, efforts are also made to reduce computing time of the present method by utilizing a p-multigrid scheme and also by solving a simplified matrix instead of a fully loaded dense matrix in the implicit matrix inversion. A superior performance of the present implicit DG method on the parallel computer using up to 128 PEs is shown in terms of readily achievable scalability and high parallel efficiency. The RANS simulation of turbulent flowfield over AGARD-B model is carried out to show the convergence property and numerical stability of the present implicit DG method for engineering applications.

AMS subject classifications: 76M10, 65M60, 65D30, 65B99

Key words: Discontinuous Galerkin method, pointwise relaxation implicit scheme, viscous compressible flow.

1 Introduction

Unstructured mesh methods are commonly used in obtaining flowfield over complete aircraft configuration because of their easiness in creating computational mesh for highly complicated geometries. These methods are also known to capture shock waves quite

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sharply using an adaptive grid refinement. In these methods, the finite volume formulation is usually chosen because the conservation laws can be rigorously fulfilled for various cell geometries. However, the spatial accuracy of these methods remains usually at most second order. The cause of this lower spatial accuracy can be attributed to the poorly reconstructed dependent variables in the computational cell. Conventional reconstruction using the cell-averaged variables in nearby cells tends to loose its accuracy for unstructured mesh particularly when cell geometries are highly skewed. In order to capture various features of complicated flowfield in practical problems, truly high order reconstruction method for unstructured mesh should be devised for finite volume methods.

Higher order reconstructions for finite volume method may be achieved for unstructured mesh by using the k-exact formulation [1] or the ENO/WENO schemes [2], though with a substantially increased amount of random memory access due to use of a wider stencil. Recently, the DG finite element method [3,4] has received attentions because of its ability in achieving higher order spatial accuracy rigorously even on unstructured mesh. In this method, instead of referring to nearby cells as in the finite volume methods, reconstruction of the dependent variables is realized with desired accuracy using the degrees of freedom (DOFs) which are introduced in each cell and evolved in time. Therefore, higher order spatial accuracy can be achieved in the DG method with minimal stencil. Indeed, it has been shown that the desired spatial accuracy could be achieved even with various cell geometries using the DG method.

The obvious shortcoming of the DG method is its extremely high computational cost. In the DG method, the dependent variables are expressed as a sum of the DOFs (expansion coefficients) multiplied with the corresponding basis functions. The number of equations to be solved in the DG method is given by a multiple of the number of dependent variables and the number of DOFs introduced in each cell. For example, the number of DOFs is 4 for 3D second order case and 20 for 3D fourth order case. Therefore, one needs to solve 100 equations for the latter case. Furthermore since the Gaussian quadrature formula is used to evaluate integrals, sufficient number of Gaussian quadrature points should be allocated both on the cell boundary and inside of the cell volume to assure numerical accuracy. This increases the number of flux evaluations and results in higher computational cost.

In order to reduce the computing cost of the DG method, it is certainly necessary to develop an implicit scheme to accelerate the convergence, particularly for those steady flow problems. It is also necessary to implement the code on vector/parallel computers. Several such attempts regarding the implicit DG method have already been studied. For example, Bassi and Rebay proposed an implicit DG method utilizing GMRES for the Navier-Stokes equations [5], Rasetarinera and Hussaini developed a matrix-free Krylov approach for the Euler equations [6], Hartmann et al. employed GMRES-Newton algorithm for the Euler and the Navier-Stokes equations [7–9], and Dolejší et al. proposed a semi-implicit DG method for the Euler and the Navier-Stokes equations [10, 11]. In developing implicit schemes, it is very important to have a flexible portability and an easier