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Droplet Collision Simulation by a Multi-Speed Lattice Boltzmann Method

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Abstract. Realization of the Shan-Chen multiphase flow lattice Boltzmann model is considered in the framework of the higher-order Galilean invariant lattices. The present multiphase lattice Boltzmann model is used in two-dimensional simulation of droplet collisions at high Weber numbers. Results are found to be in a good agreement with experimental findings.

AMS subject classifications: 76A02, 76M25, 76T10, 35Q20

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1 Introduction

1.1 The lattice Boltzmann method

The Lattice Boltzmann method (LBM) is a rapidly developing approach to computational fluid dynamics (CFD). One of its major advantages over traditional CFD is in the modelling of multiphase flows. In this paper the Shan-Chen multiphase method [1] is used to study binary droplet collisions. Numerous improvements to this model have been suggested in the literature, including the use of a different equation of state (EoS) [2], and the increase in the order of isotropy of the force term [3]. The possibilities of combining different methods will be addressed in this paper to improve stability in the droplet collision simulation. These methods are initially combined with the standard LBM. Higher order lattices, derived from an entropic viewpoint [4,5], are then considered.

The LBM originally evolved from Lattice Gas Cellular Automaton (LGCA) methods, which streamed individual particles along lattice velocities. Another starting point for

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the LBM, used here, is the Boltzmann transport equation, which describes the evolution of the density function for a gas of point like particles

$$\frac{\partial f}{\partial t}+\boldsymbol{v}\cdot\nabla_{\boldsymbol{x}} f+\boldsymbol{F}\cdot\nabla_{\boldsymbol{p}} f=\frac{\partial f}{\partial t}\Big|_{\text{coll}}, \tag{1.1}$$

where $\boldsymbol{F}$ is an external force. In this work the Bhatnagar-Gross-Krook (BGK) collision term is used on the right hand side of this equation. Discretisation of this equation, details of which can be found in many papers, including [6], leads to the LBM. The method involves the streaming of distribution functions between fixed nodes along lattice velocities $\boldsymbol{v}_i$, and then relaxing these distributions to their local equilibrium, $f_i^{\text{eq}}(\boldsymbol{x},t)$, at each lattice node (the forcing term is dropped for the time being),

$$f_i(\boldsymbol{x}+\boldsymbol{v}_i,t+1)-f_i(\boldsymbol{x},t)=-\frac{1}{\tau}\left[f_i(\boldsymbol{x},t)-f_i^{\text{eq}}(\boldsymbol{x},t)\right], \tag{1.2}$$

where τ is the relaxation time. Macroscopic quantities are recovered from moments of the distribution function

$$\rho=\sum_i f_i, \tag{1.3a}$$

$$\rho\boldsymbol{u}=\sum_i \boldsymbol{v}_i f_i. \tag{1.3b}$$

These are used to calculate the equilibrium distribution functions at each node. In two spatial dimensions, one commonly used lattice has nine velocities, (0,0), (0,$\pm$1), ($\pm$1,0) and ($\pm$1,$\pm$1), and is labeled the D2Q9 lattice. Its equilibrium distribution functions, to second order in velocity, are given by

$$f_i^{\text{eq}}=w_i\rho\left(1+\frac{\boldsymbol{v}_i\cdot\boldsymbol{u}}{c_{\text{s}}^2}+\frac{(\boldsymbol{v}_i\cdot\boldsymbol{u})^2}{2c_{\text{s}}^4}-\frac{u^2}{2c_{\text{s}}^2}\right), \tag{1.4}$$

where w_i is a weighting function, and c_{s} is the speed of sound related to the reference temperature T_0 as $c_{\text{s}}^2=T_0$. This equilibrium is designed in such a way as to reproduce the isothermal Navier-Stokes equations in the macroscopic limit, at the reference temperature $T_0=1/3$.

An alternative derivation of the LBM comes from considering entropy. Equilibrium distribution functions are derived to minimize a specified entropy function. From this entropic LBM (ELBM) a systematic method has been developed for producing stable, higher order lattices [4]. This method is outlined in Section 3.

1.2 Droplet collisions

The study of binary droplet collisions has many important applications across different scientific areas, from understanding cloud formation in climate theory, to engineering applications, such as turbine blade cooling, spray coatings and spray combustion in diesel