COMMUNICATIONS IN COMPUTATIONAL PHYSICS
Vol. **2**, No. 5, pp. 933-963

Commun. Comput. Phys.
October 2007

Non-Oscillatory Hierarchical Reconstruction for Central and Finite Volume Schemes

Yingjie Liu[1,*], Chi-Wang Shu[2], Eitan Tadmor[3] and Mengping Zhang[4]

[1] *School of Mathematics, Georgia Institute of Technology, Atlanta, GA 30332-0160, USA.*
[2] *Division of Applied Mathematics, Brown University, Providence, RI 02912, USA.*
[3] *Department of Mathematics, Institute for Physical Science and Technology, and Center of Scientific Computation and Mathematical Modeling (CSCAMM), University of Maryland, College Park, MD 20742, USA.*
[4] *Department of Mathematics, University of Science and Technology of China, Hefei, Anhui 230026, China.*

Received 30 October 2006; Accepted (in revised version) 11 January 2007

Available online 20 March 2007

Abstract. This is the continuation of the paper "Central discontinuous Galerkin methods on overlapping cells with a non-oscillatory hierarchical reconstruction" by the same authors. The hierarchical reconstruction introduced therein is applied to central schemes on overlapping cells and to finite volume schemes on non-staggered grids. This takes a new finite volume approach for approximating non-smooth solutions. A critical step for high-order finite volume schemes is to reconstruct a non-oscillatory high degree polynomial approximation in each cell out of nearby cell averages. In the paper this procedure is accomplished in two steps: first to reconstruct a high degree polynomial in each cell by using e.g., a central reconstruction, which is easy to do despite the fact that the reconstructed polynomial could be oscillatory; then to apply the hierarchical reconstruction to remove the spurious oscillations while maintaining the high resolution. All numerical computations for systems of conservation laws are performed without characteristic decomposition. In particular, we demonstrate that this new approach can generate essentially non-oscillatory solutions even for 5th-order schemes without characteristic decomposition.

AMS subject classifications: 65M06, 65M60

Key words: Central scheme, discontinuous Galerkin method, ENO scheme, finite volume scheme, MUSCL scheme, TVD scheme.

---

*Corresponding author. *Email addresses:* `yingjie@math.gatech.edu` (Y. Liu), `shu@dam.brown.edu` (C.-W. Shu), `tadmor@cscamm.umd.edu` (E. Tadmor), `mpzhang@ustc.edu.cn` (M. Zhang)

1 Introduction

Finite volume schemes are powerful numerical methods for solving nonlinear conservation laws and related equations. It evolves only cell averages of a solution over time and is locally conservative. The first-order Godunov and Lax-Friedrichs (LxF) schemes are, respectively, the forerunners for the large class of upwind and central high-resolution finite volume schemes. However, the cell average of a solution in a cell contains too little information. In order to obtain higher-order accuracy, neighboring cell averages must be used to reconstruct an approximate polynomial solution in each cell. This reconstruction procedure is the key step for many high-resolution schemes. We mention here the notable examples of the high-resolution upwind FCT, MUSCL, TVD, PPM, ENO, and WENO schemes [6, 11, 13, 14, 26, 42] and this list is far from being complete. The central scheme of Nessyahu and Tadmor (NT) [30] provides a second-order generalization of the staggered LxF scheme. It is based on the same piece-wise linear reconstructions of cell averages used with upwind schemes, yet the solution of (approximate) Riemann problems is avoided. High resolution generalizations of the NT scheme were developed since the 90s as the class of central schemes in e.g. [1, 2, 4, 16, 18, 19, 21, 22, 25, 27, 28, 35] and the list is far from being complete. The second-order MUSCL, high-order ENO and WENO reconstructions are effective non-oscillatory reconstruction methods which select the smoothest possible nearby cell averages to reconstruct the approximate polynomial solution in a cell, and can be used for uniform or unstructured meshes in multi space dimensions. In Hu and Shu [15], WENO schemes for triangular meshes are developed, and in Arminjon and St-Cyr [1], the central scheme with the MUSCL reconstruction is extended to unstructured staggered meshes. When the reconstruction order becomes higher, characteristic decomposition is usually necessary to reduce spurious oscillations for systems of conservation laws. Characteristic decomposition locally creates larger smooth area for polynomial reconstruction by separating discontinuities into different characteristic fields. Comparisons of high-order WENO and central schemes with or without characteristic decomposition are studied in Qiu and Shu [31]. As the formal order of accuracy becomes higher, e.g. 5th-order, spurious oscillations become evident for both schemes without characteristic decomposition (for the Lax problem), even though oscillations in central schemes tend to be smaller.

In a series of works by Cockburn and Shu *et al.* ([8–10] *etc*), discontinuous Galerkin (DG) methods are developed for nonlinear conservation laws and related equations. Compared to finite volume schemes, DG stores and evolves every polynomial coefficient in a cell over time. Therefore there is no need to use information in non-local cells to achieve high-order accuracy. When the solution is non-smooth, similar to finite volume schemes, DG also needs a nonlinear limiting procedure to remove spurious oscillations in order to maintain the high resolution near discontinuities. In Cockburn and Shu [8], a limiting procedure is introduced for DG which compares the variation of the polynomial solution in a cell to the variation of neighboring cell averages to detect the non-smoothness. The nonlinear part of the polynomial is truncated in the non-smooth region.