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A Pressure-Correction Scheme for Rotational Navier-Stokes Equations and Its Application to Rotating Turbulent Flows

Dinesh A. Shetty[1], Jie Shen[2,3,*], Abhilash J. Chandy[4] and Steven H. Frankel[1]

[1] *School of Mechanical Engineering, Purdue University, West Lafayette, IN 47907, USA.*
[2] *School of Mathematical Sciences, Xiamen University, Xiamen, China.*
[3] *Department of Mathematics, Purdue University, West Lafayette, IN 47907, USA.*
[4] *Department of Mechanical Engineering, University of Akron, Akron, OH 44325, USA.*

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To the memory of David Gottlieb

Abstract. The rotational incremental pressure-correction (RIPC) scheme, described in Timmermans et al. [Int. J. Numer. Methods. Fluids., 22 (1996)] and Shen et al. [Math. Comput., 73 (2003)] for non-rotational Navier-Stokes equations, is extended to rotating incompressible flows. The method is implemented in the context of a pseudo Fourier-spectral code and applied to several rotating laminar and turbulent flows. The performance of the scheme and the computational results are compared to the so-called diagonalization method (DM) developed by Morinishi et al. [Int. J. Heat. Fluid. Flow., 22 (2001)]. The RIPC predictions are in excellent agreement with the DM predictions, while being simpler to implement and computationally more efficient. The RIPC scheme is not in anyway limited to implementation in a pseudo-spectral code or periodic boundary conditions, and can be used in complex geometries and with other suitable boundary conditions.

AMS subject classifications: 52B10, 65D18, 68U05, 68U07

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1 Introduction

The ability to accurately and efficiently incorporate the effects of rotation in numerical simulations of fluid flows is important in a number of different science and engineering

*Corresponding author. *Email addresses:* `dshetty@purdue.edu` (D. A. Shetty), `shen7@purdue.edu` (J. Shen), `ac76@uakron.edu` (A. J. Chandy), `frankel@ecn.purdue.edu` (S. H. Frankel)

applications including atmospheric, oceanic, astrophysical, internal combustion engine, and turbo-machinery flows. Fundamental studies of rotating turbulence are also important for improved understanding and modeling of such flows [1]. The numerical challenge is related to the presence of the Coriolis term in the Navier-Stokes equations. Explicit treatment of this term in time limits simulations to low rotation rates and/or small time steps. This suggests that an implicit treatment of the Coriolis term is essential for stable, affordable, and realistic simulations of rotating flows. For the case of rotating homogeneous turbulence, Diagonalization Methods (DM) are often used to perform exact integration of the rotation terms [2–4]. These methods analytically integrate the Coriolis terms to provide an accurate solution. However, due to its special form, DM methods can only be applied to problems with periodic boundary conditions and is somewhat cumbersome to implement. Also, the DM method can not be easily extended to other type of boundary conditions and/or complex geometries.

In the present study, we develop a *rotational incremental pressure correction* (RIPC) method which treats the rotation term implicitly while being accurate and computationally efficient, and show the mathematical analysis of stability of the scheme. To avoid confusion, it is worth mentioning here that the word *rotational* in the name of the scheme refers to the use of the rotational form of the diffusion operator in the incompressible Navier-Stokes equations, whereas the word *rotation* refers to solid-body rotation of the flow itself. Predictions from the RIPC scheme are compared to the DM method of Morinishi et al. [4] with regard to accuracy versus efficiency. All computations in this study are performed in the context of a Fourier pseudo-spectral simulation but are in no way limited to this numerical discretization scheme or to the use of periodic boundary conditions.

The rest of the paper is organized as follows. In the next section, we introduce the RIPC scheme with rotation and prove its stability. In Section 3, we validate the accuracy and stability of the scheme against the DM scheme in [4]. We then present some direct numerical simulation (DNS) and large eddy simulation (LES) results for rotating turbulent flows in Section 4. We conclude with a few remarks in Section 5.

2 Mathematical formulation

2.1 Governing equations

The incompressible Navier-Stokes equations with rotation are given by the following:

$$\frac{\partial u_i}{\partial x_i}=0, \tag{2.1a}$$

$$\frac{\partial u_i}{\partial t}+u_j\frac{\partial u_i}{\partial x_j}=-\frac{\partial p}{\partial x_i}+\nu\frac{\partial^2 u_i}{\partial x_j \partial x_j}-(\mathbf{\Omega_j}\times\mathbf{u})_i, \tag{2.1b}$$

where u_i $(i=1,2,3)$ are the components of the velocity field $\mathbf{u}$, p is the effective pressure, ν is the kinematic viscosity, $\mathbf{\Omega}=(\Omega_1,\Omega_2,\Omega_3)$ is the rotation vector. Here we only study the