Commun. Comput. Phys.
doi: 10.4208/cicp.160410.200910a

Vol. **9**, No. 4, pp. 1056-1070
April 2011

Image Charge Method for Reaction Fields in a Hybrid Ion-Channel Model

Zhenli Xu[1,*], Wei Cai[2,3] and Xiaolin Cheng[4]

[1] *Department of Mathematics and Institute of Natural Sciences, Shanghai Jiao Tong University, Shanghai 200240, China.*
[2] *Department of Mathematics and Statistics, University of North Carolina at Charlotte, Charlotte, NC 28223, USA.*
[3] *Beijing International Center for Mathematical Research, Beijing 100871, China.*
[4] *Center for Molecular Biophysics, Oak Ridge National Laboratory, Oak Ridge, TN 37831, USA.*

Received 16 April 2010; Accepted (in revised version) 20 September 2010

Communicated by Bo Li

Available online 10 November 2010

Abstract. A multiple-image method is proposed to approximate the reaction-field potential of a source charge inside a finite length cylinder due to the electric polarization of the surrounding membrane and bulk water. When applied to a hybrid ion-channel model, this method allows a fast and accurate treatment of the electrostatic interactions of protein with membrane and solvent. To treat the channel/membrane interface boundary conditions of the electric potential, an optimization approach is used to derive image charges by fitting the reaction-field potential expressed in terms of cylindric harmonics. Meanwhile, additional image charges are introduced to satisfy the boundary conditions at the planar membrane interfaces. In the end, we convert the electrostatic interaction problem in a complex inhomogeneous system of ion channel/membrane/water into one in a homogeneous free space embedded with discrete charges (the source charge and image charges). The accuracy of this method is then validated numerically in calculating the solvation self-energy of a point charge.

AMS subject classifications: 31B05, 92C05, 65Z05

Key words: Ion channels, image charges, cylindrical harmonics, electrostatic interactions.

1 Introduction

Biological ion channels, which play a central role in controlling the appropriate electrostatic properties across the cell membrane [10, 14], are of interest in many areas of re-

*Corresponding author. *Email addresses:* xuzl@sjtu.edu.cn (Z. Xu), wcai@uncc.edu (W. Cai), chengx@ornl.gov (X. Cheng)

search such as neuroscience, cell biology, and biomedical science. To study the structural and functional properties of ion channels by dynamics simulations at the atomic scale, it is important to develop fast and accurate computational models [20, 30, 33] for treating long-ranged electrostatic interactions, in particular, to reflect the influence of the solvent and membrane.

Models employing explicit lipid and solvent molecules are impractical in many cases for large simulation systems. Implicit continuum methods (the Poisson-Boltzmann theory) provide a reasonable approximation of the electrostatic polarization effect of solvent on the structures and interactions of biomolecules in solution. For instance, the approaches [17, 19, 32, 34] based on the generalized Born theory are widely adopted in practical simulations. Hybrid explicit/implicit solvent models [28, 38] have attracted great attention for molecular simulations in aqueous solutions, which seek to take advantage of both the accuracy of explicit all-atom approaches [7, 9, 26] and the reduced cost of implicit ones [11, 15, 25, 35]. Typically, the hybrid models truncate the target system by a fixed volume which includes the solute and some explicit solvent molecules, and treat the outside solvent as a continuum medium. The benefits of such a treatment are several-fold. The primary benefit over explicit methods is the greatly reduced system size with only a small number of explicit solvent molecules to be simulated. Second, the artificial periodicity associated with most of explicit methods is avoided. Further, as the number of explicit waters is flexible, the dielectric boundary can be selected as of a regular shape, and thus analytical-based algorithms [3, 4, 36, 39] can be developed to speed up the calculations.

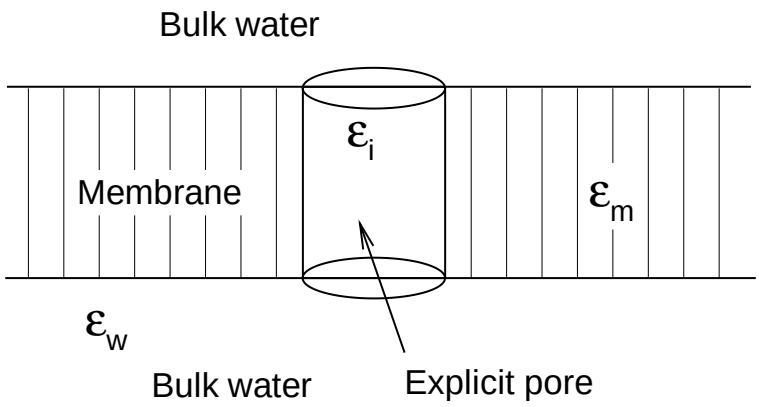

Figure 1: Schematic illustration of an ion-channel model. The molecules inside the cylindrical pore are treated explicitly at the atomic scale, while outside the pore the membrane and water are treated as homogeneous media, characterized by dielectric constants $\varepsilon_{\rm m}$ and $\varepsilon_{\rm w}$, respectively.

In the presence of a membrane, such as in the simulation of a protein ion channel embedded in a membrane lipid bilayer, however, it is challenging to develop analytical-based algorithms for the hybrid model due to the dielectric inhomogeneity of the medium. As schematically shown in Fig. 1, a hybrid ion-channel model uses a cylindrical cavity as