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Modified Baer-Nunziato Model for the Simulation of Interfaces Between Compressible Fluids

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Abstract. In this paper we proposed a modified Baer-Nunziato model for compressible multi-fluid flows, with main attention on the energy exchange between the two fluids. The proposed model consists of eleven PDEs; however, the use of the particular phase evolving variables may reduce the model to have only six PDEs. The main advantage of the model is that the Abgrall's UPV criterion on mixture velocity and pressure is satisfied without affecting either its hyperbolicity or its conservations of the two individual masses, momentum or total energy. An Lax-Friedrichs scheme is built for a particular case of the proposed model. When the two fluids in the fluid mixture are both of the linear Mie-Gruneisen type, the scheme satisfies the Abgrall's UPV criterion on mixture velocity and pressure. Numerical experiments with polytropic, barotropic, stiffened and van der Waals fluids show that the scheme is efficient and able to treat fluids characterized with quite different thermodynamics.

AMS subject classifications: 65M06, 35L65, 76N15, 76M10

Key words: Baer-Nunziato model, hyperbolicity, Abgrall's UPV criterion.

1 Introduction

The Baer-Nunziato (BN) model, see [2–5] and [8], is a two-mass, two-velocity and two-pressure diffuse model for multi-fluid flows. The model, by omitting the mass transfer, drag force, chemical reaction and convective heat exchange, has the form

$$\frac{\partial \alpha_1\hat{\rho}_1}{\partial t}+\frac{\partial \alpha_1\hat{\rho}_1 u_1}{\partial x}=0, \tag{1.1a}$$

$$\frac{\partial \alpha_1\hat{\rho}_1 u_1}{\partial t}+\frac{\partial(\alpha_1\hat{\rho}_1 u_1^2+\alpha_1\hat{p}_1)}{\partial x}=p\frac{\partial \alpha_1}{\partial x}, \tag{1.1b}$$

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$$\frac{\partial \alpha_1 \hat{\rho}_1 E_1}{\partial t} + \frac{\partial u_1(\alpha_1 \hat{\rho}_1 E_1 + \alpha_1 \hat{p}_1)}{\partial x} = pu\frac{\partial \alpha_1}{\partial x}, \tag{1.1c}$$

$$\frac{\partial \alpha_2 \hat{\rho}_2}{\partial t} + \frac{\partial \alpha_2 \hat{\rho}_2 u_2}{\partial x} = 0, \tag{1.1d}$$

$$\frac{\partial \alpha_2 \hat{\rho}_2 u_2}{\partial t} + \frac{\partial(\alpha_2 \hat{\rho}_2 u_2^2 + \alpha_2 \hat{p}_2)}{\partial x} = p\frac{\partial \alpha_2}{\partial x}, \tag{1.1e}$$

$$\frac{\partial \alpha_2 \hat{\rho}_2 E_2}{\partial t} + \frac{\partial u_2(\alpha_2 \hat{\rho}_2 E_2 + \alpha_2 \hat{p}_2)}{\partial x} = pu\frac{\partial \alpha_2}{\partial x}, \tag{1.1f}$$

$$\frac{\partial \alpha_1}{\partial t} + u\frac{\partial \alpha_1}{\partial x} = 0, \tag{1.1g}$$

where $\hat{\rho}_k$ is the phase density, u_k is the phase velocity, $\hat{p}_k$ is the phase pressure, $E_k = u_k^2/2 + \epsilon_k$ is the specific phase total energy of the kth fluid with ϵ_k the specific phase internal energy, α_k is the volume fraction satisfying the saturation condition

$$\alpha_1 + \alpha_2 = 1, \tag{1.2}$$

and finally, u and p are the averaged interfacial velocity and pressure. There are different ways to compute the interfacial velocity and pressure corresponding to different physics, e.g., in [8] they are computed as the mixture velocity,

$$u = \frac{\alpha_1 \hat{\rho}_1 u_1 + \alpha_2 \hat{\rho}_2}{\alpha_1 \hat{\rho}_1 + \alpha_2 \hat{\rho}_2}, \tag{1.3}$$

and mixture pressure,

$$p = p_1 + p_2 = \alpha_1 \hat{p}_1 + \alpha_2 \hat{p}_2, \tag{1.4}$$

respectively, where

$$\hat{p}_k = P_k(\hat{\rho}_k, \epsilon_k) \tag{1.5}$$

is the phase pressure computed by the EOS of the kth fluid as in Eq. (2.1) and $p_k = \alpha_k \hat{p}_k$ is the partial pressure contributed by the kth fluid.

The system is not conservative, and the source terms in the momentum and energy equations, called nozzling terms [3], correspond to the momentum and energy exchange between the two fluids. Although the model is complex, it is unconditionally hyperbolic and is able to treat fluids characterized by very different thermodynamics because each fluid uses its own EOS.

The Abgrall's uniform-pressure-velocity (UPV) criterion requires that a two-phase flow, uniform in pressure and velocity, must remain uniform on the same variables during its temporal evolution (see [8,9]). The BN model (1.1) and its numerical methods can meet the Abgrall's UPV criterion by requiring

$$u_1 = u_2 = \text{const}, \qquad \hat{p}_1 = \hat{p}_2 = \text{const}; \tag{1.6}$$