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A Parallel, Reconstructed Discontinuous Galerkin Method for the Compressible Flows on Arbitrary Grids

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Abstract. A reconstruction-based discontinuous Galerkin method is presented for the solution of the compressible Navier-Stokes equations on arbitrary grids. In this method, an in-cell reconstruction is used to obtain a higher-order polynomial representation of the underlying discontinuous Galerkin polynomial solution and an inter-cell reconstruction is used to obtain a continuous polynomial solution on the union of two neighboring, interface-sharing cells. The in-cell reconstruction is designed to enhance the accuracy of the discontinuous Galerkin method by increasing the order of the underlying polynomial solution. The inter-cell reconstruction is devised to remove an interface discontinuity of the solution and its derivatives and thus to provide a simple, accurate, consistent, and robust approximation to the viscous and heat fluxes in the Navier-Stokes equations. A parallel strategy is also devised for the resulting reconstruction discontinuous Galerkin method, which is based on domain partitioning and Single Program Multiple Data (SPMD) parallel programming model. The RDG method is used to compute a variety of compressible flow problems on arbitrary meshes to demonstrate its accuracy, efficiency, robustness, and versatility. The numerical results demonstrate that this RDG method is third-order accurate at a cost slightly higher than its underlying second-order DG method, at the same time providing a better performance than the third order DG method, in terms of both computing costs and storage requirements.

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1 Introduction

The discontinuous Galerkin methods [1–25] (DGM) have recently become popular for the solution of systems of conservation laws. Nowadays, they are widely used in computational fluid dynamics, computational acoustics, and computational electromagnetics. The discontinuous Galerkin methods combine two advantageous features commonly associated to finite element and finite volume methods. As in classical finite element methods, accuracy is obtained by means of high-order polynomial approximation within an element rather than by wide stencils as in the case of finite volume methods. The physics of wave propagation is, however, accounted for by solving the Riemann problems that arise from the discontinuous representation of the solution at element interfaces. In this respect, the methods are therefore similar to finite volume methods. The discontinuous Galerkin methods have many attractive features: 1) They have several useful mathematical properties with respect to conservation, stability, and convergence; 2) The method can be easily extended to higher-order ($>2^{\text{nd}}$) approximation; 3) The methods are well suited for complex geometries since they can be applied on unstructured grids. In addition, the methods can also handle non-conforming elements, where the grids are allowed to have hanging nodes; 4) The methods are highly parallelizable, as they are compact and each element is independent. Since the elements are discontinuous, and the inter-element communications are minimal, domain decomposition can be efficiently employed. The compactness also allows for structured and simplified coding for the methods; 5) They can easily handle adaptive strategies, since refining or coarsening a grid can be achieved without considering the continuity restriction commonly associated with the conforming elements. The methods allow easy implementation of *hp*-refinement, for example, the order of accuracy, or shape, can vary from element to element; 6) They have the ability to compute low Mach number flow problems without recourse to the time-preconditioning techniques normally required for the finite volume methods. In contrast to the enormous advances in the theoretical and numerical analysis of the DGM, the development of a viable, attractive, competitive, and ultimately superior DG method over the more mature and well-established second order methods is relatively an untouched area. This is mainly due to the fact that the DGM have a number of weaknesses that have yet to be addressed, before they can be robustly used to flow problems of practical interest in a complex configuration environment. In particular, there are three most challenging and unresolved issues in the DGM: a) how to efficiently discretize diffusion terms required for the Navier-Stokes equations, b) how to effectively control spurious oscillations in the presence of strong discontinuities, and c) how to develop efficient time integration schemes for time accurate and steady-state solutions. Indeed, compared to the finite element methods and finite volume methods, the DG methods require solutions of systems of equations with more unknowns for the same grids. Consequently, these methods have been recognized as expensive in terms of both computational costs and storage requirements.

DG methods are indeed a natural choice for the solution of the hyperbolic equations,