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A Pressure Relaxation Closure Model for One-Dimensional, Two-Material Lagrangian Hydrodynamics Based on the Riemann Problem

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Abstract. Despite decades of development, Lagrangian hydrodynamics of strength-free materials presents numerous open issues, even in one dimension. We focus on the problem of closing a system of equations for a two-material cell under the assumption of a single velocity model. There are several existing models and approaches, each possessing different levels of fidelity to the underlying physics and each exhibiting unique features in the computed solutions. We consider the case in which the change in heat in the constituent materials in the mixed cell is assumed equal. An instantaneous pressure equilibration model for a mixed cell can be cast as four equations in four unknowns, comprised of the updated values of the specific internal energy and the specific volume for each of the two materials in the mixed cell. The unique contribution of our approach is a physics-inspired, geometry-based model in which the updated values of the sub-cell, relaxing-toward-equilibrium constituent pressures are related to a local Riemann problem through an optimization principle. This approach couples the modeling problem of assigning sub-cell pressures to the physics associated with the local, dynamic evolution. We package our approach in the framework of a standard predictor-corrector time integration scheme. We evaluate our model using idealized, two material problems using either ideal-gas or stiffened-gas equations of state and compare these results to those computed with the method of Tipton and with corresponding pure-material calculations.

AMS subject classifications: 35L65, 65M06, 76M20, 76N15

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1 Introduction

Multi-material Lagrangian hydrodynamics of strength-free materials continues to present numerous open issues, even in one dimension. We focus on the problem of closing a system of equations for a two-material cell under the assumption of a single velocity model. We treat the constituents in these multi-material cells as distinct, i.e., we do not consider so-called "mixture" models, often associated with multi-phase flow, in which the individual species in a computational zone are modeled as fully or partially intermingled. The multi-material cells we consider invariably arise in multi-material Arbitrary Lagrangian-Eulerian (ALE) methods [13, 23], where the results of Lagrangian hydrodynamics are projected onto a new mesh during the remap phase, thereby making a Lagrangian step with a mixed cell a necessity. We consider three main design principles that govern closure models of interest. The first principle is preservation of contacts; this implies that if all materials in a mixed cell are initially at the same pressure, then that pressure does not change due to the closure model. The second principle is that of pressure equilibration; that is, after some transition time (possibly but not necessarily a single timestep), all pressures in the mixed cell equilibrate. The third principle is the exact conservation of total energy. We assume that a separate set of material properties is maintained for each material in every multi-material cell, together with the materials' volume fractions, which can be used to reconstruct material interfaces inside a mixed cell. The main challenge is to accurately assign the thermodynamic states of the individual material components together with the nodal forces that such a zone generates, pursuant to our design principles and despite a lack of detailed information about the velocity distribution within such cells. In particular, for the calculation of both the equation of state (EOS) and the resulting pressure forces, it is important that the calculation of the internal energy be accurate.

There are several existing models for this problem. In one class of methods (see, e.g., Barlow [4], Delov & Sadchikov [9], Goncharov & Yanilkin [11]), one estimates the velocity normal to the interface between materials and then approximates the change in the volume for each material, with internal energy updated separately for each material from its own pdV equation. A common pressure for a mixed cell, which is used in the momentum equation, is computed using the equation of total energy conservation. Delov & Sadchikov [9] and Goncharov & Yanilkin [11] introduce an exchange of internal energy between the materials inside a mixed cell, thereby allowing some freedom in the definition of the common pressure. Other multi-material models impose either instantaneous pressure equilibration (such as that of Lagoutière [18] and Després & Lagoutière [10]) or ascribe an implicit mechanism for pressure relaxation (such as described by Tipton [30] and summarized by Shashkov [28]).

We restrict our attention to the approach in which the change in heat in the constituent materials in the mixed cell is posited to be equal, following Lagoutière [18] and Després & Lagoutière [10]. Under this assumption, the mixed-cell model can be cast as four equations in four unknowns, consisting of the updated values of the specific internal energy and the specific volume for each of the two materials in the mixed cell. A solution to this