A Discontinuous Galerkin Method for Ideal Two-Fluid Plasma Equations

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Abstract. A discontinuous Galerkin method for the ideal 5 moment two-fluid plasma system is presented. The method uses a second or third order discontinuous Galerkin spatial discretization and a third order TVD Runge-Kutta time stepping scheme. The method is benchmarked against an analytic solution of a dispersive electron acoustic square pulse as well as the two-fluid electromagnetic shock [1] and existing numerical solutions to the GEM challenge magnetic reconnection problem [2]. The algorithm can be generalized to arbitrary geometries and three dimensions. An approach to maintaining small gauge errors based on error propagation is suggested.

1 Introduction

Fusion power promises to be a safe, efficient and environmentally friendly energy source. Controlled fusion power concepts have been under investigation for decades, the vast majority of these concepts require an intimate understanding of plasma physics to determine the stability and confinement properties. Numerical plasma physics has proved extremely valuable in deciphering experimental data and predicting the behavior of plasma experiments. Many plasma fluid models, and in particular the full two-fluid plasma model, have received very little attention from the numerical plasma physics community.

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This work describes an advanced algorithm for the ideal 5-moment two-fluid plasma system.

To solve problems in plasma physics and to gain physical intuition of plasma phenomena a hierarchy of classical plasma models have been developed. The most fundamental continuum plasma model is the Vlasov model which eliminates individual particles in favor of a continuous distribution function. This model is six dimensional as the distribution function is a function of both position and velocity. The Vlasov model can be re-written as an equivalent system that consists of an infinite number of moment equations. A reduction of the Vlasov model can then be obtained by truncating this infinite series. Assuming scalar pressure and setting the heat tensor and higher moments to zero produces the 5 moment truncation of the Vlasov model. This model is known as the ideal 5 moment two-fluid plasma model, and will be discussed in this paper. Asymptotic approximations of this two-fluid system produce a series of increasingly simpler fluid models including two-fluid MHD (Magnetohydrodynamics), Hall MHD and then the ideal MHD models.

The main benefit of a fluid model over the Vlasov model is the reduced dimensionality from 6 dimensions to 3 dimensions. Physics is lost in this reduction, but an enormous amount of physics relevant to fusion and spacecraft propulsion remains in the fluid description. Ideal MHD has been extremely successful in explaining large scale instabilities in such devices as the Z-pinch, spheromak and tokamak [3, 4]. Unfortunately there are many regimes where the description is invalid and where it fails to explain the observed phenomena. An example of this includes ion demagnetization which is important in Field Reversed Configurations [5] and Hall thrusters. Hall MHD addresses both these issues but fails to describe other plasma phenomena such as the demagnetization of electrons in regions of low magnetic field which is important in collisionless reconnection. The two-fluid MHD approach adds terms such as electron inertia which is an important mechanism for breaking the frozen in flux condition for electrons as it acts as a "dissipation" mechanism in the absence of resistivity [6]. The quasi-neutrality condition still constrains the electron and ion motions, to allow complete independence of electron and ion motion the quasi-neutrality condition must be relaxed; the result is the ideal two-fluid plasma system.

Two-fluid effects are important in the generation of turbulence through microinstabilities. Most plasmas are turbulent at some scale, however the simplest fluid model, ideal MHD, describes plasmas physics that is more or less laminar where the two-fluid model produces turbulent phenomena. This can be explained in part by the fluid description of electrons. In a two-fluid model both the electrons and the ions may become unstable independently. In particular, electrons carry most of the current in an MHD plasma. This current may produce a large amount of differential motion in the electron fluid when magnetic field gradients are present even if the plasma is in a static MHD equilibrium. The generation of microturbulence through processes such as the lower hybrid drift instability and the modified two-stream instability may be important in both Z-pinch and theta-pinch plasmas. These instabilities are frequently cited as sources of anomalous re-