Journal of Computational Mathematics
Vol.38, No.5, 2020, 797–826.
http://www.global-sci.org/jcm
doi:10.4208/jcm.2002-m2019-0192

ACCURATE AND EFFICIENT IMAGE RECONSTRUCTION FROM MULTIPLE MEASUREMENTS OF FOURIER SAMPLES*

T. Scarnati
Air Force Research Laboratory, WPAFB, OH 45433, USA
Email: theresa.scarnati.1@us.af.mil
Anne Gelb
Dartmouth College, Hanover, NH 03755, USA
Email: annegelb@math.dartmouth.edu

Abstract

Several problems in imaging acquire multiple measurement vectors (MMVs) of Fourier samples for the same underlying scene. Image recovery techniques from MMVs aim to exploit the joint sparsity across the measurements in the sparse domain. This is typically accomplished by extending the use of ℓ_1 regularization of the sparse domain in the single measurement vector (SMV) case to using $\ell_{2,1}$ regularization so that the "jointness" can be accounted for. Although effective, the approach is inherently coupled and therefore computationally inefficient. The method also does not consider current approaches in the SMV case that use spatially varying weighted ℓ_1 regularization term. The recently introduced variance based joint sparsity (VBJS) recovery method uses the variance across the measurements in the sparse domain to produce a weighted MMV method that is more accurate and more efficient than the standard $\ell_{2,1}$ approach. The efficiency is due to the decoupling of the measurement vectors, with the increased accuracy resulting from the spatially varying weight. Motivated by these results, this paper introduces a new technique to even further reduce computational cost by eliminating the requirement to first approximate the underlying image in order to construct the weights. Eliminating this preprocessing step moreover reduces the amount of information lost from the data, so that our method is more accurate. Numerical examples provided in the paper verify these benefits.

Mathematics subject classification: 49N45, 68U10, 65T99.
Key words: Multiple measurement vectors, Joint sparsity, Weighted ℓ_1, Edge detection, Fourier data.

1. Introduction

There are many applications for which multiple (indirect) measurement vectors (MMVs) of data are acquired that represent the same underlying scene, e.g., a signal or image. The scene is typically recovered using each *single* measurement vector (SMV) separately, that is without exploiting any measurement redundancies or inter-signal correlations between measurements. In the case of noisy or incomplete data sources, compressed sensing (CS) algorithms employing ℓ_1 regularization are often used so that the results are optimally sparse in some domain, e.g., the gradient or wavelet domain [1, 2]. Some processing may follow after these individual reconstructions to infer information about the underlying scene, but there is always loss of information due to the individual processing of the indirect data, [3–5].

* Received August 23, 2019 / Revised version received November 11, 2019 / Accepted February 12, 2020 / Published online July 6, 2020 /

More recently a number of methods have been developed to exploit the *joint sparsity* across the MMVs of the same scene via $\ell_{2,1}$ regularization. One can view $\ell_{2,1}$ regularization as an extension of the usual ℓ_1 regularization approach. In particular, $\ell_{2,1}$ minimization enforces the overlapping sparse support of functions in a projected domain, and reconstructs multiple approximations of the underlying function. This technique was developed and thoroughly analyzed throughout [6–12] and references therein, but results depend on careful hand tuning of the parameters. The methods cited above were designed to produce a more accurate collection of estimates but typically do not produce a single, representative reconstruction of the underlying scene.

There have also been several developments for CS algorithms in the SMV case using spatially adaptive weighted ℓ_1 regularization. The idea here is to enforce more regularization in regions where the underlying signal or image is presumably zero (without value) in the sparsity domain, and by contrast less penalty at locations in the sparsity domain corresponding to non-zero values. These algorithms thus ostensibly improve the accuracy and robustness of classic ℓ_1 regularization techniques by eliminating the need to hand tune sensitive regularization parameters. Most of these algorithms solve a sequence of weighted ℓ_1 minimization problems, with weights iteratively updated at each step [13–18]. In some cases, [5, 19–21], a well chosen weight can eliminate the need to use ℓ_1 regularization, and the optimization can be performed using the much more computationally efficient ℓ_2 regularization. Due to noise in the data and an inaccurate approximation of the solution in the sparse domain, the weighted ℓ_p, $p = 1, 2$, approach is not always effective, however, and in some cases may even yield *worse* results than the traditional (uniformly weighted) approach. This issue is discussed in more detail in Section 3.

A few investigations have combined the ideas of weighted regularization with those that exploit joint sparsity for MMV, [11, 12, 22–24]. Noteably, the variance based joint sparsity (VBJS) technique was developed in [23, 24] to use the *variance* between measurements in the sparsity domain to determine a spatially adaptive weight to use in the aforementioned weighted ℓ_p, $p = 1, 2$ regularization. VBJS proved to be robust with respect to noise, and in [23] it was also shown to be effective even when some measurements from multiple data sources contained misleading or incorrect information. In [25], this VBJS approach was adapted for synthetic aperture radar (SAR) image formation.

In its original form, the VBJS method used initial reconstructions from each SMV (typically using the CS framework) *before* calculating the variance between measurements in the sparse domain. In other words, the initial CS approximations were simply improved upon by exploiting the joint sparsity properties. This initial processing was done regardless of how the data were acquired. Though accurate and generalizable to any linear or non-linear forward model, the method becomes increasingly computationally complex as the number of measurement vectors increases – a problem we describe in more detail in Section 5. Moreover, such initial processing causes information loss, especially in low resolution and low signal-to-noise (SNR) environments. Thus we are motivated to improve both the performance and efficiency of VBJS by eliminating the need to pre-process the reconstruction given certain types of data acquisition methods.

As will be demonstrated in this investigation, in applications where data are acquired as Fourier samples, the joint sparsity property can be exploited *without* having to first perform multiple reconstructions of the underlying image or signal. Our specific approach is to determine an accurate approximation of the edges (internal boundaries) of the underlying signal or image directly from the acquired Fourier data using the concentration factor (CF) edge detection