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Review Article

Model Meets Deep Learning in Image Inverse Problems

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Abstract. Image inverse problem aims to reconstruct or restore high-quality images from observed samples or degraded images, with wide applications in imaging sciences. The traditional methods rely on mathematical models to invert the process of image sensing or degradation. But these methods require good design of image prior or regularizer that is hard to be hand-crafted. In recent years, deep learning has been introduced to image inverse problems by learning to invert image sensing or degradation process. In this paper, we will review a new trend of methods for image inverse problem that combines the imaging/degradation model with deep learning approach. These methods are typically designed by unrolling some optimization algorithms or statistical inference algorithms into deep neural networks. The ideas combining deep learning and models are also emerging in other fields such as PDE, control, etc. We will also summarize and present perspectives along this research direction.

AMS subject classifications: 00-XX

Key words: Image inverse problem, model-driven deep learning, statistical model, optimization model.

1 Introduction

Image inverse problem [1] attempts to reconstruct / restore high-quality images from a few observed samples or degraded images captured by different imaging equipments. It has wide applications in medical imaging [2], compressive sensing image reconstruction [3], natural image restoration [4]. It is a fundamental and challenging task because it requires to well regularize the inverse process to have a better solution in the image

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space. The specific tasks that involve image inverse problems include image denoising, image deblur, image dehaze, image restoration, image inpainting, super-resolution, etc. The research on image inverse problem has been popularized for a long history, based on traditional signal processing methods [5], regularization-based methods [6–8], and deep learning methods [2, 9, 10].

1.1 Traditional methods

Traditionally, image inverse problems are mainly tackled by different model-based methods. The essential contributions of these methods are the design of different regularizers or priors from various perspectives, e.g., regularized variational methods, sparse representation methods and statistical methods. The *variational methods* are based on energy models taking image as continuous function, and regularized by total variation (TV) [4, 6, 11], generalized total variation (GTV) [7, 12], non-local regularizers [8, 13] or wavelet domain regularization [14, 15]. *Sparse representation-based method* assumes that the image or image patches can be represented by a sparse combination of basis in a dictionary [8, 16–20]. *Statistical methods* in image inverse problem generally restore / reconstruct image by maximum a posteriori estimation using a Bayesian framework, and different natural image priors are designed based on statistical models such as Gaussian scale mixture [21], Markov random field [22] or conditional random field [23].

These traditional mathematical models are commonly built based on physical mechanism of imaging and image degradation process, with the expert-designed image prior and regularizer to constrain the image space. Due to the modeling of psychical mechanism, these models are explainable and without relying on a large set of training data as the deep learning approach that will be introduced as follows.

1.2 Deep learning methods

Deep learning [24] approach has been recognized as a state-of-the-art tool in artificial intelligence with wide applications in face recognition [25], machine translation [26], chess and Go competition [27], and medical image analysis [28].

In recent years, deep neural networks have been introduced to solving image inverse problems [1, 2, 29]. These deep learning methods directly learn a mapping from the degraded image to the high-quality image taking advantage of the nonlinearity and high capacity of deep neural network. Various networks have been devised for image super-resolution, denoising [30], inpainting [30], medical image reconstruction. For image super-resolution, Dong et al. [9] firstly uses a deep convolutional neural network (CNN) to learn an end-to-end mapping between the low/high-resolution images, and then Kim et al. [31] uses a very deep CNN achieving higher accuracy. To reconstruct or restore realistic images, [32] proposed to use generative adversarial network (SRGAN) and perceptual loss to introduce visually realistic textures [33, 34]. Then, [35] combined the per-pixel loss and perceptual loss for Image super-resolution task. For medical image