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Domination in Generalized Cayley Graph of Commutative Rings

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Abstract. Let R be a commutative ring with identity and n be a natural number. The generalized Cayley graph of R, denoted by Γ_R^n, is the graph whose vertex set is $R^n \setminus \{0\}$ and two distinct vertices X and Y are adjacent if and only if there exists an $n \times n$ lower triangular matrix A over R whose entries on the main diagonal are non-zero such that $AX^T = Y^T$ or $AY^T = X^T$, where for a matrix B, B^T is the matrix transpose of B. In this paper, we give some basic properties of Γ_R^n and we determine the domination parameters of Γ_R^n.

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1 Introduction

The theory of Cayley graphs has grown into a substantial branch of algebraic graph theory in the last few decades. The concept of Cayley graph was introduced by Aurther Cayley in 1878 to explain the abstract groups which are described by a set of generators. Cayley graphs of groups have been extensively studied and some interesting results have been obtained (see [3]). Also, the Cayley graphs of semi groups have been considered by some authors (see [5,8–14,16]).

Let R be a commutative ring with identity. Sharma and Bhatwadekar [18] defined the comaximal graph on R, denoted by $\Gamma(R)$, with all elements of R being the vertices of $\Gamma(R)$, where two distinct vertices a and b are adjacent if and only if $aR+bR=R$. In [15,19], the authors considered a subgraph $\Gamma_2(R)$ of $\Gamma(R)$ consisting of non unit elements of R, and investigated several properties of the comaximal graph. Also the comaximal graph

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of a non-commutative ring was defined and investigated in [20]. Note that the subgraph of the undirected Cayley graph $\overline{Cay}(R^*,R^*)$ consisting of non-unit vertices is a subgraph of the complement of the comaximal graph $\Gamma_2(R)$. Moreover, the two subgraphs of the undirected Cayley graph $\overline{Cay}(R^*,R^*)$ and the comaximal graph $\Gamma(R)$ consisting of unit elements of R are isomorphic. In view of this, Khashyarmanesh et al. [1] introduced and characterized the rings in terms of the genus and crosscap numbers of the generalized Cayley graph Γ_R^n of a commutative ring R. For a natural number n and a commutative ring R with identity element, we associate a simple graph, denoted by Γ_R^n, with $R^n\setminus\{0\}$ as the vertex set and two distinct vertices X and Y are adjacent if and only if there exists an $n\times n$ lower triangular matrix A over R whose entries on the main diagonal are non-zero such that $AX^T=Y^T$ or $AY^T=X^T$, where for a matrix B, B^T is the matrix transpose of B. In case $n=1$, the resulting graph is the undirected graph $\overline{Cay}(R^*,R^*)$. They determined the clique number of the graph which is always greater than or equal to $|U(R)||R|^{n-1}$, where $U(R)$ is the set of all units in R. In this paper, we obtain the domination number of generalized Cayley graph associated with rings.

Let G be a graph with vertex set V. A subset D of V is called a dominating set of G if every vertex in $V\setminus D$ is adjacent to at least one vertex in D. The domination number of G, denoted by $\gamma(G)$, is the minimum cardinality of a dominating set of G. A subset D of V is called total dominating set of G if every vertex in V is adjacent to at least one vertex in D. The total domination number of G, denoted by $\gamma_c(G)$, is the minimum cardinality of a total dominating set of G. A dominating set D is called a connected dominating set if the induced subgraph $\langle D\rangle$ is connected. The connected domination number $\gamma_c(G)$ of a graph G equals the minimum cardinality of a connected dominating set in G. A dominating set D is called an independent dominating set of G if no two vertices of D are adjacent in G. The minimum cardinality of an independent dominating set of G is the independent domination number $\gamma_i(G)$. For basic definitions on graphs, we refer to [4,6,17].

A ring $(R,\mathfrak{m})$ is called local if it has a unique maximal ideal $\mathfrak{m}$. We note that $J(R)$ is the Jacobson radical of R and $Reg(R)$ is the set of regular elements of R. For any set X, let X^* denote the non-zero elements of X. We denote the ring of integers modulo n by $\mathbb{Z}_n$, the field with q elements by $\mathbb{F}_q$. For basic definitions on rings, one may refer [2,7]. Throughout this paper, we assume that R is a commutative ring with identity and $n>1$.

The following results are useful in the subsequent sections.

Lemma 1.1 ([1]). *Let $X=(x_1,x_2,\cdots,x_n)\in R^n\setminus\{0\}$ be a vertex whose first component is a unit and $Y=(y_1,y_2,\cdots,y_n)\in R^n\setminus\{0\}$ be a vertex whose first component is non-zero. Then X and Y are adjacent in Γ_R^n. Further, the induced subgraph of all vertices whose first components are units is a complete graph.*

Theorem 1.1 ([1]). *If R is an integral domain, then Γ_R^n is disconnected. Moreover, Γ_R^n has n components and every component is a refinement of a star. Hence, if R is a field, then Γ_R^n has n complete components.*

2 Domination number of Γ_R^n

We determine the domination number of Γ_R^n. For each i with $1\le i\le n$, we use the notation $\mathcal{C}_i$ to denote the set of all vertices whose first non-zero elements are in the i^{th} place, E_i is a vertex whose i^{th} element is 1 and the other elements are zero and

$$V(\Gamma_R^n)=\bigcup_{i=1}^{n}\mathcal{C}_i.$$

Theorem 2.1. *If R is a field, then Γ_R^n is perfect. For i, $(1\le i\le n)$,*

$$deg(X)=|R^*||R|^{n-i}-1$$

for all $X\in\mathcal{C}_i$.

Proof. By Theorem 1.1, Γ_R^n has n complete components, each of its components is induced by $\mathcal{C}_i$ for $1\le i\le n$ and so the clique number and chromatic number of any induced subgraph of Γ_R^n are equal and hence Γ_R^n is perfect. Note that $|\mathcal{C}_i|=|R^*||R|^{n-i}$ for every $i=1,2,\cdots,n$. Hence, for any $X\in\mathcal{C}_i$, $deg(X)=|\mathcal{C}_i|-1$. $\square$

Theorem 2.2. *Let (R,m) be a local ring and $m\neq\{0\}$ is a principal ideal. Then $\gamma(\Gamma_R^n)=1$.*

Proof. Since R is local, so $ann(x)=m$ for some $x\in m$ and $x\neq 0$. As m is principal, therefore $m=\langle a\rangle$ for some $a\in m^*$. Consider a vertex $X=(x_1,x_2,\cdots,x_n)\in V(\Gamma_R^n)$ such that $x_1=a$ and x_2 is an unit in R. Let $Y=(y_1,y_2,\cdots,y_n)$ be any vertex of Γ_R^n and $X\neq Y$. We have the following cases to consider.

Case 1. If $y_1\in U(R)$, then consider an $n\times n$ lower triangular matrix $A=[a_{ij}]$ over R whose entries satisfy the following properties $a_{11}=x_1y_1^{-1}$, $a_{ii}=1$ for $i>1$, $a_{i1}=y_1^{-1}(x_i-y_i)$ for $i>1$, $a_{ij}=0$ for $i\neq j$ and $j>1$. Therefore,

$$AY^T=\begin{pmatrix} x_1y_1^{-1} & 0 & 0 & \cdots & 0\\ y_1^{-1}(x_2-y_2) & 1 & 0 & \cdots & 0\\ y_1^{-1}(x_3-y_3) & 0 & 1 & \cdots & 0\\ \vdots & \vdots & \vdots & & \vdots\\ y_1^{-1}(x_n-y_n) & 0 & 0 & \cdots & 1\end{pmatrix}\times\begin{pmatrix}y_1\\ y_2\\ y_3\\ \vdots\\ y_n\end{pmatrix}=\begin{pmatrix}x_1\\ x_2\\ x_3\\ \vdots\\ x_n\end{pmatrix}=X^T.$$

Hence X and Y are adjacent in Γ_R^n.

Case 2(a). Suppose $y_1\in m^*$ and $y_2\neq 0$. Since $m^*=\langle a\rangle$, so $y_1=ra$ for some $r\in R^*$. Consider an $n\times n$ lower triangular matrix $A=[a_{ij}]$ over R with entries on the main-diagonal to be non-zero and the entries satisfies the following properties $a_{11}=r$, $a_{ii}=1$ for $i>2$, $a_{22}=x_2^{-1}y_2$,

$a_{i2}=x_2^{-1}(-x_i+y_i)$ for $i>2$, $a_{ij}=0$ for $i\neq j$ and $j\neq 2$. Therefore,

$$AX^T=\begin{pmatrix} r & 0 & 0 & \cdots & 0 \\ 0 & x_2^{-1}y_2 & 0 & \cdots & 0 \\ 0 & x_2^{-1}(-x_3+y_3) & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & x_2^{-1}(-x_n+y_n) & 0 & \cdots & 1 \end{pmatrix}\times\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{pmatrix}=\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_n \end{pmatrix}=Y^T.$$

Hence X and Y are adjacent in Γ_R^n.

Case 2(b). Suppose $y_2=0$. Then we construct an $n\times n$ lower triangular matrix $A=[a_{ij}]$ over R with non-zero diagonal entries such that the entries satisfy the following properties $a_{11}=r$, $a_{ii}=1$ for $i>2$, $a_{21}=1$, $a_{22}=-x_2^{-1}x_1$, $a_{i2}=x_2^{-1}(-x_i+y_i)$ for $i>2$, $a_{ij}=0$ for $i\neq j$ and $j\neq 2$. Thus,

$$AX^T=\begin{pmatrix} r & 0 & 0 & \cdots & 0 \\ 1 & -x_2^{-1}x_1 & 0 & \cdots & 0 \\ 0 & x_2^{-1}(-x_3+y_3) & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & x_2^{-1}(-x_n+y_n) & 0 & \cdots & 1 \end{pmatrix}\times\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{pmatrix}=\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_n \end{pmatrix}=Y^T.$$

Hence X and Y are adjacent in Γ_R^n.

Case 3(a). If $y_1=0$ and $y_2\neq 0$, then consider an $n\times n$ lower triangular matrix $A=[a_{ij}]$ over R whose entries satisfy the following properties $a_{11}=x$, $a_{ii}=1$ for $i>2$, $a_{22}=x_2^{-1}y_2$, $a_{i2}=x_2^{-1}(-x_i+y_i)$ for $i>2$, $a_{ij}=0$ for $i\neq j$ and $j\neq 2$. Therefore,

$$AX^T=\begin{pmatrix} x & 0 & 0 & \cdots & 0 \\ 0 & x_2^{-1}y_2 & 0 & \cdots & 0 \\ 0 & x_2^{-1}(-x_3+y_3) & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & x_2^{-1}(-x_n+y_n) & 0 & \cdots & 1 \end{pmatrix}\times\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{pmatrix}=\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_n \end{pmatrix}=Y^T.$$

Hence X and Y are adjacent in Γ_R^n.

Case 3(b). Suppose $y_1=0$ and $y_2=0$. Then we construct an $n\times n$ lower triangular matrix $A=[a_{ij}]$ over R whose entries satisfy the following properties $a_{11}=x$, $a_{ii}=1$, for $i>2$, $a_{21}=1$, $a_{22}=-x_2^{-1}x_1$, $a_{i2}=x_2^{-1}(-x_i+y_i)$ for $i>2$, $a_{ij}=0$ for $i\neq j$ and $j\neq 2$. Thus,

$$AX^T=\begin{pmatrix} x & 0 & 0 & \cdots & 0 \\ 1 & -x_2^{-1}x_1 & 0 & \cdots & 0 \\ 0 & x_2^{-1}(-x_3+y_3) & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & x_2^{-1}(-x_n+y_n) & 0 & \cdots & 1 \end{pmatrix}\times\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{pmatrix}=\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_n \end{pmatrix}=Y^T.$$

Hence X and Y are adjacent in Γ_R^n. From the above three cases, it follows that X is adjacent to all other vertices of Γ_R^n and so $\gamma(\Gamma_R^n)=1$. $\square$

Theorem 2.3. *Let R be a ring which is not an integral domain and is other than the local ring whose maximal ideal is principal. Then*

$$\gamma(\Gamma_R^n)=2=\gamma_c(\Gamma_R^n).$$

Proof. Since R is not an integral domain, there exist non-zero elements a and b in R such that $ab=0$. Let

$$D=\{X=(x_1,x_2,\cdots,x_n),\ Y=(y_1,y_2,\cdots,y_n)\},$$

where $x_1,y_2\in U(R),y_1=a\in Z^*(R)$. Let $Z=(z_1,z_2,\cdots,z_n)$ be any vertex in Γ_R^n. We consider the following cases.

Case 1. Suppose that $z_1\neq 0$. Then we construct a $n\times n$ lower triangular matrix $A=[a_{ij}]$ over R whose entries satisfy the following properties $a_{11}=x_1^{-1}z_1$, $a_{ii}=1$ for $i>1$, $a_{i1}=x_1^{-1}(z_i-x_i)$ for $i>1$, $a_{ij}=0$ for $i\neq j$ and $j>1$. Thus, we have

$$AX^T=\begin{pmatrix} x_1^{-1}z_1 & 0 & 0 & \cdots & 0\\ x_1^{-1}(z_2-x_2) & 1 & 0 & \cdots & 0\\ x_1^{-1}(z_3-x_3) & 0 & 1 & \cdots & 0\\ \vdots & \vdots & \vdots & & \vdots\\ x_1^{-1}(z_n-x_n) & 0 & 0 & \cdots & 1 \end{pmatrix}\times\begin{pmatrix} x_1\\ x_2\\ x_3\\ \vdots\\ x_n\end{pmatrix}=\begin{pmatrix} z_1\\ z_2\\ z_3\\ \vdots\\ z_n\end{pmatrix}=Z^T.$$

Hence X is adjacent to all the vertices whose first component is non-zero.

Case 2. Suppose $z_1=0$ and $z_2\neq 0$. We can construct the lower triangular matrix $A=[a_{ij}]$ over R with non-zero diagonal entries as follows $a_{i1}=b$ for $i\geq 1$, $a_{22}=y_2^{-1}\,z_2$, $a_{ii}=1$ for $i>2$, $a_{i2}=y_2^{-1}\,(-y_i+z_i)$ for $i>2$, $a_{ij}=0$, for $i\neq j$ and $j>2$. Thus,

$$AY^T=\begin{pmatrix} b & 0 & 0 & 0 & \cdots & 0\\ b & y_2^{-1}z_2 & 0 & 0 & \cdots & 0\\ b & y_2^{-1}(z_3-y_3) & 1 & 0 & \cdots & 0\\ \vdots & \vdots & \vdots & \vdots & & \vdots\\ b & y_2^{-1}(z_n-y_n) & 0 & 0 & \cdots & 1 \end{pmatrix}\times\begin{pmatrix} y_1\\ y_2\\ y_3\\ \vdots\\ y_n\end{pmatrix}=\begin{pmatrix} z_1\\ z_2\\ z_3\\ \vdots\\ z_n\end{pmatrix}=Z^T.$$

Therefore Y is adjacent to all the vertices whose first non-zero component is in the second place.

Case 3. Suppose $z_1=z_2=0$. Now, we consider the lower triangular matrix $A=[a_{ij}]$ over R with non-zero diagonal entries, where $a_{i1}=b$ for $i\neq 2$, $a_{21}=-1$, $a_{22}=y_1y_2^{-1}$, $a_{ii}=1$ for

$i>2$, $a_{i2}=y_2^{-1}\,(-y_i+z_i)$ for $i>2$, $a_{ij}=0$ for $i\neq j$ and $j>2$. Thus,

$$AY^T=\begin{pmatrix} b & 0 & 0 & 0 & \cdots & 0\\ -1 & y_1y_2^{-1} & 0 & 0 & \cdots & 0\\ b & y_2^{-1}(-y_3+z_3) & 1 & 0 & \cdots & 0\\ \vdots & \vdots & \vdots & \vdots & & \vdots\\ b & y_2^{-1}(-y_n+z_n) & 0 & 0 & \cdots & 1\end{pmatrix}\times\begin{pmatrix} y_1\\ y_2\\ y_3\\ \vdots\\ y_n\end{pmatrix}=\begin{pmatrix} z_1\\ z_2\\ z_3\\ \vdots\\ z_n\end{pmatrix}=Z^T.$$

Hence Y is adjacent all the vertices whose first component is zero. Hence, from the above three cases, it follows that D is a dominating set of Γ_R^n and so $\gamma(\Gamma_R^n)\leq 2$.

Now, let $\gamma(\Gamma_R^n)=1$. Then there exists a vertex $X=(x_1,x_2,\cdots,x_n)\in\Gamma_R^n$ such that $\{X\}$ is a dominating set of Γ_R^n. Let $Y=(y_1,y_2,\cdots,y_n)$ be any vertex in Γ_R^n. Consider first the case $x_1\in U(R)$. Then consider Y such that $y_1=0$. It can be easily verified that X is not adjacent to Y in Γ_R^n, a contradiction.

Suppose $x_1\in Z(R)$. Then consider Y such that $y_1\in Reg(R)$. If X and Y are adjacent, then we can find a lower triangular matrix $A=[a_{ij}]$ over R whose entries on the main diagonal are non-zero such that $AX^T=Y^T$ or $AY^T=X^T$. If $AX^T=Y^T$, then $a_{11}x_1=y_1$ which is impossible. If $AY^T=X^T$, then $a_{11}y_1=x_1$ which is again impossible.

If $x_1\in Reg(R)$, then consider Y such that $y_1\in Z(R)$. Suppose X and Y are adjacent, we get a similar contradiction as in the above case. Thus $\gamma(\Gamma_R^n)=2$.

By Lemma 1.1, X and Y are adjacent in Γ_R^n and hence $\gamma_c(\Gamma_R^n)=2$. □

Theorem 2.4. *If R is an integral domain, then $\gamma(\Gamma_R^n)=n$.*

Proof. Since R is an integral domain, by Theorem 1.2, Γ_R^n is disconnected. Also, Γ_R^n has n components and every component is a refinement of a star graph with center E_i for $1\leq i\leq n$. Let $D=\{E_1,E_2,\cdots,E_n\}$. Then D is a dominating set and hence $\gamma(\Gamma_R^n)=n$. □

If $R\cong\mathbb{Z}_2$, then as in the proof of Theorem 2.1, Γ_R^n has n complete components induced by $\mathcal{C}_i$ for $1\leq i\leq n$ and $|\mathcal{C}_n|=1$. Hence $\gamma_t(\Gamma_R^n)$ does not exist.

Theorem 2.5. *If R is a commutative ring and $R\ncong\mathbb{Z}_2$, then $\gamma_t(\Gamma_R^n)=2$ or $2n$.*

Proof. Case 1. Suppose R is not an integral domain. Then there exist non-zero elements a and b such that $ab=0$. Let $D=\{X,Z\}$, where $X=(x_1,x_2,\cdots,x_n)$, $x_1\in U(R)$ and $Z=(z_1,z_2,\cdots,z_n)$, $z_1=a$, $z_2\in U(R)$. As in the proof of Theorem 2.3, D is a dominating set. Since $x_1\in U(R)$, by Lemma 1.1, X and Z are adjacent and so $\gamma_t(\Gamma_R^n)=2$.

Case 2. Suppose R is an integral domain. Then, by Theorem 1.2, Γ_R^n is disconnected. Also, Γ_R^n has n components G_i for $1\leq i\leq n$ and every component is a refinement of a star graph with center $E_i\in G_i$ for $1\leq i\leq n$. Let $D=\{E_1,F_1,E_2,F_2,\cdots,E_n,F_n : F_i\in G_i,\ F_i\neq E_i,\ 1\leq i\leq n\}$. Then D is a total dominating set with minimum cardinality and hence $\gamma_t(\Gamma_R^n)=2n$. □

In the next theorem, we use the notation $X_i=(x_{i1},x_{i2},\cdots,x_{in})$ for a vertex in Γ_R^n.

Theorem 2.6. *If R be a ring, then* $\gamma_i(\Gamma_R^n) \leq n$.

Proof. Let $D = \{X_1, X_2, \cdots, X_n\}$, where

$$X_k = (\underbrace{0,0,\cdots,0}_{i=k-1}, x_{ki}, x_{k(i+1)}, \cdots, x_{kn}) \in \mathcal{C}_k$$

for $k,i = 1,2,\cdots,n$ and the first non-zero component is a unit. Now, we have to prove that D is dominating set of Γ_R^n. Let Y be any vertex in Γ_R^n with $Y \notin D$. Since

$$V(\Gamma_R^n) = \bigcup_{i=1}^{n} \mathcal{C}_i,$$

so $Y \in \mathcal{C}_k$ for some k. By definition of $\mathcal{C}_k$,

$$Y = (\underbrace{0,0,\cdots,0}_{i=k-1}, y_{ki}, y_{k(i+1)}, \cdots, y_{kn})$$

for $i = 1,2,\cdots,n$, is a vertex in Γ_R^n.

Consider the $n \times n$ lower triangular matrix $A = [a_{ij}]$ over R whose entries on the main diagonal are non-zero and satisfies the following conditions, $a_{ii} = 1$ for $i \neq k$, $a_{ii} = x_{k_i}^{-1} y_{k_i}$ for $i = k$, $a_{ij} = x_{k_j}^{-1}(y_{j_i} - x_{j_i})$ for $i > k$, $j = k$, $a_{ij} = 0$ for $i \neq j$, $j \neq k$. Then $AX_k^T = Y^T$ and so X_k and Y are adjacent in Γ_R^n. It is clear that X_i and X_j are not adjacent in Γ_R^n for any $i \neq j$. Since the vertex set of Γ_R^n is the disjoint union of $\mathcal{C}_i$, therefore $\gamma_i(\Gamma_R^n) \leq n$. □

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